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Review Article

Analysis of Potential Fluid Flow, Harmonic Function and its Application

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Abstract: This project is aimed at showing the application of potential flow in fluid mechanics. It has been shown so far in this project that for an incompressible irrotational and inviscid flow, the vorticity vector is the curl of the flow velocity, which gives a potential kind of flow. It was also demonstrated in this project that the harmonic function is a solution of the Laplace equation, it gives in fluid mechanics to the potential kind of flow, which is also called the irrotational flow. The irrotationality of a potential flow is due to the curl of the gradient of a scala, always being equal to zero. The flow described by this model is a potential field, the velocity potential function and the determination of the velocity component from its scalar function. i.e. by differentiating the stream function with respect to the given coordinates are described in this project. A description of the reduction of the equations of motion for "ideal" (irrotational, incompressible and inviscid) flow to a single equation. Viz the laplace equation is provided in some details.

Keywords: Potential Flow, Harmonic Functions, Navier-Stokes Equation, Fluid Mechanics, Laplace Equation

INTRODUCTION

The concept of potential flow is derived from the Navier-Stokes equation, named after Claude-Louis Navier and George Gabriel Stokes, describing the motion of fluid

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substances. The equation arises from applying Newton's second law of fluid motion, together with the assumption that the stress in fluid is the same of a diffusing viscous term (proportional to the gradient of velocity) and a pressure term, hence describing viscous flow (White, 2022)

The equations are useful because they describe the physics of many things of academic and economic interest. They may be used to model the weather, ocean current, water flows in a pipe, and air flows around a wing(Kindly et al; 2021, Chanson 2022).

The Navier-Stokes equation in their full and simplified form helps with the designs of air craft and cars, the study of blood flows, the designs of power stations, the analysis of political and many other things.

A solution of Navia Stokes equation is called a velocity field or a flow field, which is a description of the velocity of the field at a given point in space or time. Once the velocity field is solved for other quantities of interest (such as, flow rate or drag force) may be found.

The Navier-Stokes equations are non-linear partial differential equations. The non-linearity is due to the convective acceleration, which is an acceleration associated with the change in velocity over position. As a result, a potential flow is characterized by a non-rotational velocity field, which is a valid approximation for several applications.

a. Objectives of the Study

i. To show that the stream function and the velocity potential satisfies the Laplace equation.

ii. To understand the basic principles behind potential flow.

iii. To apply the concept of mass momentum and energy conservation to flows.

iv. To study analytical solutions to variety of simplified problems.

v. To develop and improve my ability to do research work independently.

b. Significance of the Study

A study of this nature is significant as it helps to analyse the principles behind potential flow and how they can be applicable to aerodynamics, marine hydrodynamics, air craft design, ground after waves and electroosmotic flow.

1.3 Statement of the Problem

A major problem encountered in some different fields in physical sciences is the one o analyzing potential flow, harmonic functions and their applications. This research is attempt at addressing this problem.

1.4. General Concept

1.4.1 A Description of Naiver-Stokes Equation

The Naiver-Stokes equation begins with an application of Newton's second law. Conservation of momentum (often alongside mass and energy conservation) being written for an ordinary portion of the fluid. The general form of the equation of fluid motion is (Naiver-Stokes equation) given below.

$$\rho \left(\frac{\partial V}{\partial t} + V \bullet \nabla V \right) = -\nabla \rho + V \cdot T$$

where:

V-denotes the fluid velocity

p-denotes the fluid density

p- denotes the pressure

F- denotes body force (per unit volume) acting on the fluid

Equation (i) is often written using the material derivative $\frac{\Delta v}{\Delta t}$ making it more apparent that it is a statement of Newton's second law.

$$\rho \frac{\nabla V}{\nabla t} = \nabla p + \nabla \cdot T + F$$

The left-hand side of equations (ii) describes the acceleration and may be composed of time dependence or convective effect (also the effect of non-inertial coordinate, if present).

The right-hand side of the equation is in effect a summation of the body forces (such as gravity) and divergence of stress (pressure and shear stress). A significant feature of the Naiver-Stoke equation is the presence of convective acceleration. The effect of time independent acceleration of a fluid with respect to space, while individual fluid particles are indeed experiencing time dependent acceleration. The convective acceleration of a fluid flow field is a spatial effect, with one example being fluid speeding up in a nozzle.

Convective acceleration is represented by the non-linear quantity $V \cdot \nabla V$ which may be represented as $(V \cdot \nabla)$ or as $(\nabla \cdot V)$ with the tensor derivative of the velocity vector. Both interpretations give the same result, independent of the coordinate system provided it is interpreted as the coordinate derivative.

The Naiver-Stokes equation result from the following assumption on the derivative stress tensor.

i. The derivative stress varnishes for a fluid at rest.
ii. The fluid is assumed to be isotropic as valid for glass and simple liquid and consequently is an isotropic tensor. Furthermore, since the deviatomic stress tensor is symmetric, it turns out that it can be expressed in terms od scaler dynamic viscosities.

iii. In the Navier-Stikes equation, the deviatomic stress is expressed as the product of the tensor gradient of the flow velocities with a viscosity tensor.

The Naiver-Stroke equation is strictly a statement of the conservation of momentum. In order to fully describe the fluid flow, more information is needed. These additional information may include boundary data (or an equation of state). Regardless of the flow, assumption of a statement of the conversation of mass is generally necessary. This is achieved through the mass continuity of equation given its most general form as:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho V) = 0$$

1.5.2 Incompressible flow of a Newtonian fluid

Anderson covers compressible flow and shock waves, then links classical theory to modern CFD methods for solving Navier-Stokes equations. Useful for connecting theory to computational practice (Anderson, 2021).

The assumption of incompressibility rules out the possibility of sounds or shaves water to occur. So this simplification is not useful if this phenomena of interest. The incompressible flow assumption typically holds well, even when dealing with a compressible flood Such as air at room temperature at low mark (0. 3). Taking the incompressible flow assumption into account and assuming constant viscosity, the Naiver-Stroke equation will lead in vector form as:

$$\rho\left(\frac{\partial V}{\partial t} + V \bullet \nabla V\right) = -\nabla\rho + \mu\nabla^2 V + F$$

Here, F represents other body forces per unit volume, such as gravity or centrifugal force. They shear stress term becomes the useful quality. $\mu\nabla^{2V}$(Is vector Lapalcian) When the fluid is assumed incompressible homogeneous and Newtonian where ρ is the (constant) dynamic viscosity. It is worth observing the meaning of each term (compared to Cauchy momentum equation.)

$$\rho\ \frac{\partial V}{\partial t} + V \bullet \nabla V = -\nabla\rho + \mu\nabla^2 V + F$$

Where $\frac{\partial V}{\partial t}$is unsteadily acceleration, V . ∇V is the convection acceleration, - ∇p is the pressure gradient, $\mu\nabla^2 V$ is the viscosity, and F represent other body forces.

Note that only the convective terms are non-linear for incompressible Newtonian flow. The convective acceleration is an acceleration caused by a (possible steady) change in velocity over position. For example, the speeding up of fluid entering a conversing nozzle through individual fluid particles are being accelerated and one under-steady motion, the flow fluid (a velocity distribution) will not necessarily be time dependent.

Another observation is that the viscosity of the velocity fluid (interpreted here as the difference between the velocity at a point and the mean velocity in a small volume around this implies that for a Newtonian fluid viscosity operates in a diffusion of momentum, in much the same way as the diffusion of heat seen in the heat equation, which is also involved in the Laplacian). If the temperature effects are neglected, the only other equation (apart from initial boundary condition) needed is the mass compressibility, the density of a fluid parcel is constant and its follows that the continuity equation will simplify to this more specifically a statement of the conservation of volume.

These equations are commonly used in a three coordinate system, Cartesian cylindrical and spherical, while the Cartesian equation seems to follow directly from the vector equation. The vector form of the Naiver-Stoke equation involves some tensor Calculus which means that writing it in other coordinate system is not as simple as doing so far for scalar equation (heat equation, for example).

1.4.3 Cartesian Coordinate

Writing the vector equation explicitly:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \frac{\partial \rho}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} +\right)\rho g x$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \frac{\partial \rho}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} +\right)\rho g y$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \frac{\partial \rho}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \rho g z$$

Note that gravity has been accounted for as a body force, and the value of gx, gy and gz will depend on the orientation of gravity with respect to the chosen set of coordinate.

The continuity equation read:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

When the flow is incompressible it does not change for any fluid parcel and its material

$$\frac{\Delta \rho}{\Delta t} = 0$$

1.4.4 The Derivative Variables

The continuity equation is reduced to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

The velocity component (the dependent variables to be solved for) is typically named u, v and w. this system of four equations comprises the most commonly used and studied form through comparatively more compact than other representation. This is still a non-linear system of partial differential equation for which solutions are difficult to obtain.

The continuity equation reduces to Laplace equation; that is, $\nabla.\ V = O$ reduces to:

$$\nabla 2\emptyset = \frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} + \frac{\partial^2 \emptyset}{\partial z^2} = 0$$

1.4.5 No Slip Condition

In fluid dynamics the no slip condition for viscous fluid state that at a solid boundary, the fluid have zero velocity relative to the boundary. The fluid velocity at all fluid solid boundaries is equal to that of the solid boundary. Conceptually one can think of the most outer molecules of fluid as sticks to the surface past which it flows because the solution is prescribed at a given location. This is an example of derichilet boundary condition.

1.4.6 Physical Justification of no Slip Condition

Particles close to a surface do not move along with a flow when adhesion is stronger than cohesion.

As an exception, as with most engineering approximation, the no slip condition does not always hold reality. For example, at very highpressure (example, at very high altitude) even when the continuum approximation still holds. They may be so few molecules near the surface that they bounce along down the surface. A common approximation for fluid slip is:

$$U - U_{wall} = \beta \frac{du}{dn}$$

Where n is the coordinate normal to the wall, β is called the slip length for an ideal gas, the slip length is often approximated as β = 1.15L, wher L is the free path.

Some lightly hydrophobic surface has been observed to have a non-zero but no scale length, while the no slip condition is used to universally in modeling various flows. It is sometime neglected in favor of the non-penetration condition (where the fluid velocity normal to the wall is set to the wall velocity in the direction, but the fluid velocity parallel to the wall is set to the wall is unrestricted). In elementary analysis of inviscid flow, where the effect of boundary layers is neglected the no slip condition poses a problem in viscous flow theory at contact line places where an interface between two fluid boundary condition implies that the position of the contact line does not move, which is not observed in reality.

Analysis of the moving constant lines with the no slip condition result in infinite stress that cannot be integrated over, the rate of movement of the contact line is believed to be independent of the angle the contact line makes.

PROBLEMS FORMULATION

From the application of harmonic function in fluid mechanics, we want to consider the fluid velocity *q = (u,v,w)* at time *t*. further, suppose that at the considered instance *t*, there exist a scalar function $\emptyset\ (x, y, z, t)$, uniform through the entire field of flow, and such that:

$$-\ d\emptyset = udx + vdy + wdx \qquad (1)$$

That is,

$$\frac{d\emptyset}{dx}dx + \frac{d\emptyset}{dy}dy + \frac{d\emptyset}{dz}dz = udx + vdy + wdx \qquad (2)$$

Then the expression at the right hand side of the equation is an exact differential equation. As in exact differential, we have:

$$u =- \frac{d\emptyset}{dx},\ v =- \frac{d\emptyset}{dy},\ w =- \frac{d\emptyset}{dz} \quad (3)$$

Then,

$$q =- \nabla\emptyset =- grad\emptyset \quad (4)$$

Which is then called the velocity potential. The negative sign in equation (4) is a convection. It ensures that the flow takes place from the higher to the lower potentials.

The necessary and significant condition for equation (4) to hold is:

$$\nabla x q = 0,\ this\ is\ Curl\ q = 0 \quad (5)$$

Where q is the velocity vector; *u, v* and *w* are component of the velocity vector in the *x, y* and *z* plane; ∅ is the scalar function; *x, y, z,* and *t* are components of scalar function respectively

Equation (5) can also be written as:

$$\begin{pmatrix} i & j & k \\ u & v & w \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{pmatrix} = i\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + j\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + k\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \quad (6)$$

This is called the potential flow of the surface. But

$$\emptyset(x, y, z, t) = constant \quad (7)$$

Are called the equipotentials.

The streamlines:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \quad (8)$$

are cut at right angle by the surface given by the differential equation:

$$udx + vdy + wdz = 0 \quad (9)$$

And the condition for the existence of such orthogonal surface is that condition (9) may posses a solution of the form (7). At the considered instance, the analytic conditions is:

$$u\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + v\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + w\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \quad (10)$$

when the velocity potential exists, equation (3) holds.

Then

$$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} =- \frac{\partial^2}{\partial y \partial z} + \frac{\partial^2}{\partial z \partial y} = 0 \quad (11)$$

That is,

$$\left.\begin{aligned}\frac{\partial u}{\partial z} &= \frac{\partial w}{\partial x}\\ \frac{\partial v}{\partial x} &= \frac{\partial u}{\partial y}\end{aligned}\right\} \quad (12)$$

Using condition (11) and (12) we find out that the condition is satisfied. Hence, surface exist, which cuts across the streamline's orthogonally. We conclude that at all point of the fluid of flow, the equi-potentials are cut orthogonally by the streamlines.

When equation (5) holds, the flow is known as the potential kind; that is, potential flow which is also known as irrotational. For such flow the field q is conservative.

The equation of continuity of an incompressible fluid is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (13)$$

Suppose that the fluid moves irrotationally then the velocity potential $\emptyset$ exist, such that:

$$u = -\frac{\partial \emptyset}{\partial x},\ v = -\frac{\partial \emptyset}{\partial y}, w = \frac{\partial \emptyset}{\partial z} \quad (14)$$

Applying (13) and equation (14) reduces to:

$$\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} + \frac{\partial^2 \emptyset}{\partial z^2} = 0 \quad (15)$$

Showing that $\emptyset\ is\ a$ harmonic function satisfying the Laplace equation $\nabla^2 \emptyset = 0$, where,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = 0 \quad (16)$$

In continuum mechanics the vorticity vector is a pseudovector field that describes the local spinning of a continuum near some point (the tendency of something to rotate) it is an important quantity in the dynamical theory of fluids, and provides a convenient framework for understanding a variety of complex flow phenomena such as the formation and motion of vortex rings.

Mathematically the vorticity Ω is the curl of the flow velocity $\vec{V}$.

$$\Omega = \nabla\, x\, \vec{V} \quad (17)$$

Where $\vec{V}$gives the fluid velocity,

$V_x V_y V_t$ are components of the velocity vector.

The vorticity of a three dimensional flow is a pseudovector fluid, usually denoted by $\vec{\Omega}$ defined as the curl of the velocity field $\vec{V}$ describing the continuum motion. In Cartesian coordinates:

$$\Omega = \nabla\, x\, \vec{V} = \left(\frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z}\right) X\ (V_x V_y V_z)$$

$$= \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \quad \frac{\partial V_z}{\partial z} - \frac{\partial V_z}{\partial x} \quad \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) \quad (18)$$

If $\Omega_{x,}$ $\Omega_{y,}$ $\Omega_{z,}$ be the component of Ω in Cartesian coordinate.

Then equation (18) becomes

$$\Omega_{x,} = \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}$$

$$\Omega_{y,} = \frac{\partial V_x}{\partial z} - \frac{\partial V_y}{\partial x}$$

$$\Omega_{z,} = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \quad (19)$$

In words the vorticity tells how the velocity vector changes when one moves by an infinitesimal distance in a direction perpendicular to it.

In a two dimensional flow where the velocity is independent of the z-coordinate and has a z-component, the vorticity vector is always parallel to the z axis and therefore can be expressed as a scalar fluid, multiplied by a constant unit vector $\hat{Z}$:

$$\vec{\Omega} = \nabla x \vec{V} = \left(\frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \right) \mathrm{X} \, (V_x V_y V_z)$$

$$= \left(\frac{\partial V_x}{\partial x} - \frac{\partial V_y}{\partial y} \right) \dot{Z} \quad (20)$$

The vorticity is also related to the flow's circulation (line integral of the velocity) along a close path by the (classical) stokes theorem. Namely, for an infinitesimal surface element C with normal direction $\vec{n}$ and area dA, the circulation dT along the parameter of C is the dot product $\vec{\Omega}$ $(\vec{n}dA)$where $\vec{\Omega}$ is the vorticity at the centre of C.

SOLUTION PROCEDURE

Analysis of the Problem

Example 1:

An incompressible flow fluid is characterized by the stream function

$$\psi = 3ax^2\mathrm{y} - ay^3$$

Show that the flow is irrotational

Solution 1

The given flow is a two-dimensional flow. That is, $\psi = \psi\ (x,y)$ for the flow to be irrotational, we must have:

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

But $u = \frac{\partial \psi}{\partial x}$ and $v = -\frac{\partial \psi}{\partial x}$

Thus, we write the irrotational condition in the form:

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} = -\nabla^2 \psi = 0$$

Evaluating $V^{2\psi}$, we obtain:

$$V^{2\psi} = \frac{\partial^2}{\partial x^2}\left(3ax^2y - ay^3\right) + \frac{\partial^2}{\partial y^2}\left(3ax^2y - ay^3\right) = 6ay - 6ay = 0$$

Thus, the flow is irrotational.

Example 2:

Consider the flow fluid given by $\psi = ax^2 - ay^2$ where $a = 1\ sec^{-1}$.

(i) Show that the flow is irrotational

(ii) Determine the velocity potential

(iii) Show that the line of constant ψ and ∅ are orthogonal.

Solution 2:

(i) The given flow is two-dimensional since $\psi = \psi\,(x, y)$. For the flow to be irrotational, we must have:

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

Where $u = \frac{\partial \psi}{\partial x}\ and\ v = -\frac{\partial \psi}{\partial x}$

Thus, we write the irrotationality condition in the form:

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi}{\partial y^2} = -\nabla^2 \Psi = 0$$

Evaluating $\nabla^2 \Psi$, we obtain:

$$\nabla^2 \Psi, = \frac{\partial^2}{\partial x^2}\left(ax^2 - ay^2\right) = \frac{\partial^2}{\partial y^2}\left(ax^2 - ay^2\right) = 2a - 2a = 0$$

Since $-\nabla^2 \Psi = 0$, we conclude that the flow is irrotational.

(ii) $u = \frac{\partial \Psi}{\partial x} = -2ay\ and\ v = -\frac{\partial \Psi}{\partial x} = -2ax$

The velocity component can also be written in terms of the velocity potential.

$$u = -\frac{\partial \emptyset}{\partial x}\ and\ v = -\frac{\partial \emptyset}{\partial y}$$

Thus,

$$\frac{\partial \emptyset}{\partial x} = 2ay \qquad (1)$$

$$\frac{\partial \emptyset}{\partial y} = 2ax \tag{2}$$

Integrating equation (1) with respect to *x,* we obtain:

$$\emptyset = 2axy + f(y) \tag{3}$$

$$\frac{\partial \emptyset}{\partial y} = 2ax + f^{'}(y) \tag{4}$$

Comparing (2) with (4), we see that:

$f^{'}(y) = 0 \Rightarrow f(y) =$ Constant

Putting this into equation (3), we have:

$\emptyset = 2axy + constant$

(iii) For Ψ = Constant, we have:

$$\partial\Psi = \frac{\partial\Psi}{\partial x}dx + \frac{\partial\Psi}{\partial y}dy = 0$$

$$\Rightarrow 2axdx - 2aydy = 0$$

$$\Rightarrow \frac{\partial y}{\partial x}\ \Psi = constant = \frac{x}{y}$$

For $\emptyset$ = constant, we have:

$$\partial\emptyset = \frac{\partial\emptyset}{\partial x}dx + \frac{\partial\emptyset}{\partial y}dy = 0$$

$$\Rightarrow 2aydx - 2axdy = 0$$

$$\Rightarrow \frac{\partial y}{\partial x}\ \Psi = constant = -\frac{y}{x}$$

Observe that the product of the two slopes gives -1.

4.3 Result

From the analysis of potential flow in fluid mechanics, a rotational flow occurs when the curl of velocity of the fluid is everywhere zero. That is, ∇ x *V*=0.

From the analysis of the given problems in chapter four, the result shows that for a flow to be irrotational it must satisfy the condition:

$$\nabla^{2\Psi} = 0$$

Analyzing the flow field given above, we obtain the result to be zero everywhere satisfying the condition$\nabla^{2\Psi} = 0$, which shows that the flow is irrotational.

SUMMARY, CONCLUSION, AND RECOMMENDATIONS

Summary.

In a nutshell, this project discuses the application of potential fluid flow and all the application of harmonic function in fluid mechanics. The general concept of Naiver-Stokes equation, potential fluid flow in mechanics and its rotation where shown with the definition of harmonic. And also, it was clearly shown that the harmonic function is a solution of Laplace equation.

Conclusion

It has been shown so far in this project that harmonic function can be applied in fluid mechanics, and that the harmonic functions is a solution of the Laplacian equation. It was also shown that the vorticity equation of fluid dynamics describes the evolution of the vorticity Ω of a particle of a fluid as it moves with its flow i.e. the vorticity tells us how the velocity vector changes when one moves by an infinitesimal distance in a direction perpendicular to its.

Recommendations

This research has considered the problem of analyzing potential fluid flow and its application in fluid mechanics. The investigation also revealed that the harmonic function is a solution of the Laplace equation it gives in fluid mechanics to the potential kind of flow, which is also called the irrotational flow.

It was also demonstrated that curl of the potential is always equal to zoro. It is therefore recommended that further investigation may be made in an irrotational flow field where one can use either velocity potential or stream function to characterize the flow such that both must fulfil Laplace equation.

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