Journal of Contemporary Academic Research and Methodologies

An Online Peer Reviewed / Refereed Journal
Volume 1 | Issue 5 | June 2026
Website: www.journal.jcarm.in

Original Research Article

SATISFICING UNDER FIRE: LEXICOGRAPHIC GOAL PROGRAMMING FOR PARAMILITARY WORKFORCE OPTIMIZATION IN RESOURCE-CONSTRAINED ENVIRONMENTS

Uchenwa Linus Okafor[1], Udoh Israel Jacob[2*], Abam Ayeni Omnini[3], Christopher Eraye Michael[4], Obia Gopeh Inyang[5], Blessing George Abam[6]

[1] Department of Mathematical Sciences, Nigerian Defence Academy, Kaduna.
[2] Applied Mathematics & Simulation Advanced Research Centre (AMSARC) Sheda Science & Technology Complex (SHESTCO), Abuja.
[3] Department of Mathematics and Statistics, Federal University, Lafia, Nigeria.
[4] Department of Criminology and Security Studies, Federal University, Lafia, Nigeria.
[5] University of Calabar Library, Cross River State, Nigeria.
[6] Nigerian Security and Civil Defence Corps (NSCDC), Headquarters, Abuja.

Correspondence should be addressed to Udoh Israel Jacob: ij.udoh@shestco.gov.ng

Article No: 057 | **Accepted**: 15 June 2026 | **Published:** 10 July 2026

Abstract: Paramilitary workforce planning in resource-constrained environments presents unique optimization challenges where traditional linear programming fails to accommodate conflicting operational mandates. This study introduces a lexicographic goal programming (LGP) framework, specifically calibrated for the Nigerian Security and Civil Defence Corps (NSCDC), addressing critical gaps in security sector operations research. We formulate and validate three pre-emptive GP models: multi-cadre training allocation under fiscal volatility, simultaneous armed/unarmed deployment scheduling, and weapon training time distribution under regulatory constraints. Methodologically, we integrate Hannan's efficiency test to eliminate dominated solutions and propose a novel stochastic extension accounting for instructor absenteeism; addressing a persistent limitation in deterministic security scheduling literature. Analysis of 54 comparative studies yields a taxonomy distinguishing security-sector GP applications from civilian counterparts across ten operational dimensions. Results reveal that current weapon training regulations are stochastically infeasible at 95% reliability, while training allocation models identify a 12% fiscal buffer previously obscured by heuristic scheduling practices. Shadow price analysis demonstrates that junior cadre

 https://doi.org/10.5281/zenodo.21368943

quotas, not budgetary limits, constitute the true binding constraint in training optimization. Beyond numerical results, we provided LINGO/GAMS code templates, extended simplex tableaux, and a four-phase change management framework for adopting quantitative methods within hierarchical paramilitary cultures. This research establishes LGP as both an optimization engine and diagnostic instrument for evidence-based security administration, offering transferable methodologies for Global South paramilitary organizations navigating expanding mandates amid fiscal austerity. The framework enables commanders to quantify trade-offs transparently, advocate for resources with analytical credibility, and align workforce planning with strategic priorities through rigorous satisficing rather than unattainable optimality..

Keywords: Lexicographic Goal Programming, Paramilitary Workforce Optimization, Satisficing; Efficiency Testing, Stochastic Programming; NSCDC, Resource-Constrained Environments, Multi-Criteria Decision Making.

1.0 INTRODUCTION

Security workforce planning (SWP) operates in a high-stakes environment characterized by resource scarcity and competing operational mandates. Traditional Linear Programming (LP) optimizes a single objective function subject to hard constraints. However, security administration involves satisficing multiple, often incommensurable goals, such as minimizing cost while maximizing coverage and maintaining morale, where rigid constraints are operationally unrealistic (Charnes et al., 1955; Ignizio, 1976). This study establishes the operational imperative for Goal Programming (GP) within paramilitary contexts, tracing its evolution from a constrained regression technique to a strategic decision-support architecture. We examine why traditional LP is structurally inadequate for multi-mandate security agencies, and frame the Nigerian Security and Civil Defence Corps' (NSCDC) post-2019 challenges as a canonical multi-criteria decision-making problem. *If budgetary volatility and mandate expansion are permanent rather than cyclical, can security administration afford to continue optimizing for a single objective while ignoring the trade-offs that dictate actual operational capacity?* GP, the oldest approach in Multi-Criteria Decision Making (MCDM), resolves this by minimizing deviations from aspiration levels rather than optimizing a single metric. While GP has been applied to nurse scheduling (Mohammadian et al., 2018) and airline crew pairing (Ilkay, 2022), its application to paramilitary workforce planning remains under-theoritized.

1.1 The Imperative for MCDM in Paramilitary Workforce Planning

SWP represents one of the most complex optimization challenges in public sector administration. Unlike commercial entities where profit maximization serves as a singular unifying objective, paramilitary organizations such as the NSCDC must navigate a multi-dimensional objective space characterized by conflicting mandates: operational readiness, fiscal austerity, personnel welfare, regulatory compliance, and community safety. Traditional LP, while mathematically elegant, is fundamentally ill-suited for this environment. LP requires hard constraints and a single objective function; yet, in security administration, constraints are frequently "soft" (aspirational rather than absolute), and objectives are incommensurable measured in Naira, man-hours, casualty rates, and training certifications simultaneously (Ignizio, 1976; Romero, 1991).

GP emerges as the preeminent analytical framework for addressing these complexities. As the oldest approach within the MCDM, GP shifts the paradigm from “optimization” to “satisficing”. Rather than seeking an elusive global optimum that may be operationally infeasible, GP minimizes the weighted sum of deviations from predetermined aspiration levels across multiple goals (Charnes et al., 1955). This paper applies GP specifically to the NSCDC workforce management context, demonstrating how lexicographic and weighted GP variants can resolve deployment scheduling, training allocation, and resource distribution problems that have historically relied on heuristic or ad-hoc decision-making.

1.2 Historical Evolution of GP in Security Studies

The theoretical foundations of GP were established by Charnes, Cooper, and Ferguson (1955) in their seminal work on constrained regression analysis for executive compensation. However, it was not until the 1970s and 1980s that GP matured into a distinct MCDM methodology through the contributions of Ignizio (1976), Lee (1972), and Romero (1991). While early applications focused on production planning and financial portfolio management, the security sector has been a persistent, if under-documented, beneficiary of GP methodologies.

Historically, GP applications in security have evolved through three distinct phases. The first phase (1970–1990) focused primarily on police patrol scheduling and shift rostering; treating officer hours as divisible commodities subject to union contracts and coverage requirements. These models were largely deterministic and single-location specific. The second phase (1990–2010) integrated multi-objective frameworks that acknowledged the tension between cost efficiency and service quality. Notably, Zeleny (1981) and Hannan (1980) addressed critical limitations regarding dominated solutions and inefficiency; providing the theoretical safeguards necessary for high-stakes security applications where suboptimal solutions could compromise public safety.

The third phase (2010–present) has seen GP applied to increasingly complex, multi-agency security environments. Ozder et al. (2018) demonstrated GP’s efficacy in assigning security personnel to heterogeneous locations with varying threat profiles; moving beyond simple shift scheduling to strategic asset allocation. Similarly, Mohammadian et al. (2018) and Sharma and Jain (2016) adapted GP for healthcare and emergency services scheduling; establishing precedence for handling personnel preferences alongside organizational mandates - a critical consideration for maintaining morale in paramilitary forces. Ilkay (2022) further advanced the field through bi-objective programming for airline crew pairing; saving millions in operational costs while satisfying regulatory rest requirements. Despite these advances, a significant gap remains in the application of GP to African paramilitary contexts, particularly those operating under severe resource constraints and volatile security environments like Nigeria. This paper directly addresses that lacuna.

1.3 Post-2019 Challenges in NSCDC Workforce Management

The NSCDC operates within a uniquely constrained environment that distinguishes it from Western security agencies often featured in GP literature. Since 2019, the Corps has faced an inflection point driven by four converging pressures that render traditional workforce planning obsolete. First, fiscal volatility and budgetary uncertainty have become structural rather than cyclical. With national security budgets frequently subjected to mid-year adjustments and delayed releases, NSCDC State Commands cannot rely on fixed-cost assumptions. Hard-constraint LP models fail catastrophically under these conditions; declaring feasible operational plans "infeasible" due to minor budget overruns. GP's soft constraint architecture allows commandants to model budgetary targets as prioritized goals rather than rigid boundaries; enabling continued operations even at reduced funding levels. Secondly, the expansion of statutory mandates without commensurate personnel increase has created acute deployment dilemmas. The NSCDC's responsibilities now encompass critical infrastructure protection, counter-terrorism support, disaster response, and civil order maintenance. Each mandate carries distinct staffing ratios, skill requirements, and equipment needs. Allocating limited personnel across these competing priorities requires explicit trade-off modeling that only GP can provide.

Thirdly, training pipeline bottlenecks have emerged as a systemic vulnerability. With junior, middle, and senior cadres requiring different skill sets (Skills A, B, and C as detailed in Section 5), and training facilities operating below capacity due to scheduling inefficiencies, the Corps faces a paradox: unfilled training slots coexist with untrained personnel awaiting deployment. The multi-cadre training allocation problem formulated in this paper addresses this inefficiency directly. Fourth, personnel welfare and retention concerns have intensified in post-2019 NSCDC operations. Excessive overtime, unpredictable deployments, and inadequate rest periods contribute to burnout and attrition. Modern security workforce GP models must incorporate welfare metrics as explicit goals, not merely as afterthoughts to ensure sustainable force generation. This aligns with recent scholarship emphasizing "human-centric" scheduling in high-stress professions (Mohammadian et al., 2018; Sharma & Jain, 2016).

1.4 Research Objectives and Contributions

This paper makes four primary contributions to the intersection of operations research (OR) and security workforce management:

i. **Context-Specific Model Formulation:** We develop three validated GP models tailored to NSCDC operational realities: (a) multi-cadre training allocation with budgetary and quota constraints; (b) simultaneous armed/unarmed deployment across dual operations; and (c) weapon training time allocation under regulatory minimums.
ii. **Theoretical Rigor Enhancement:** We address the dominated solution problem identified by Romero (1991) and Zeleny (1981) through explicit efficiency testing protocols, ensuring all proposed solutions are Pareto-optimal.
iii. **Numerical Verification and Correction:** We re-examine and correct arithmetic inconsistencies present in prior working papers, providing verified simplex tableaux and sensitivity analyses for reproducibility.

iv. **Policy Translation Framework:** We bridge the gap between mathematical abstraction and command-level decision-making, offering interpretable deviational variable analyses that quantify exactly "how much" each goal is compromised under resource scarcity.

1.5 Organization of the Paper

The remainder of this article is structured as follows. Section 2 presents the theoretical framework, including formal definitions of pre-emptive and non-preemptive GP, proofs of equivalence under specific conditions, and the Hannan-Masud efficiency test. Section 3 provides an extended literature review synthesizing over 50 studies across security, healthcare, and transportation scheduling, identifying specific gaps this paper addresses. Section 4 details the model formulation methodology, including linearization techniques and priority structure design. Section 5 presents the three numerical case studies with verified solutions, sensitivity analyses, and computational implementation notes. Section 6 discusses policy implications for NSCDC State Commands, including software requirements and change management considerations.

Finally, Section 7 concludes with limitations and future research directions, including stochastic GP extensions for uncertain threat environments. Appendices provide LINGO/GAMS code templates, extended simplex tableaux, and mathematical proofs for reader verification. Through this comprehensive treatment, we aim to establish GP not merely as an academic exercise, but as an operational necessity for modern paramilitary workforce management in resource-constrained environments.

2.0 THEORETICAL FRAMEWORK

Before modeling operational realities, we must formalize the mathematical architecture that transforms conflicting aspirations into solvable systems. This section constructs the theoretical scaffolding of GP, defining deviational variables, establishing lexicographic priority structures, and proving the conditions under which weighted and pre-emptive formulations converge. We rigorously address the dominated solution paradox and embed Hannan's efficiency test into the security scheduling paradigm, ensuring that mathematical optimality translates to operational Pareto efficiency. *When does a mathematically optimal schedule become operationally inefficient, and how can efficiency testing be institutionalized rather than treated as an academic afterthought?*

2.1 Foundational Axioms of GP

GP represents a fundamental departure from classical optimization. While LP seeks to optimize a single objective function over a feasible region defined by hard constraints, GP operates on the Simonian principle of "satisficing" - seeking solutions that achieve acceptable levels across multiple, often conflicting, aspirations (Simon, 1955). The contemporary literature correctly identifies GP as an extension of LP; however, it is more accurately characterized as a distinct subclass of MCDM where the utility function is replaced by an achievement function based on deviational variables.

Let $x \in R_{+}^{n}$ be the vector of decision variables. For each goal $i = 1,..., m$, we define a target aspiration level g_i. The relationship between the realized value $f_i(x)$ and the target is governed by the fundamental goal constraint:

$$f_i(x) + d_i^- - d_i^+ = g_i \tag{2.0.0}$$

where d_i^-, $d_i^+ \geq 0$ are the negative and positive deviational variables, representing underachievement and overachievement respectively. A critical property, often assumed but rarely proven in applied literature, is the complementarity condition: $d_i^- \cdot d_i^+ = 0 \ \forall i$

Theorem 2.1 (Complementarity in Basic Solutions). *In any basic feasible solution to a linear GP problem where the objective function minimizes a weighted sum of deviations with strictly positive weights, at most one of d_i^- or d_i^+ can be basic (non-zero) for any given goal i.*

Proof. Consider the column vectors corresponding to d_i^- and d_i^+ in the simplex tableau. By definition, these columns are identical except for sign (+1 and -1). Thus, they are linearly dependent. In any basis matrix B, linearly dependent columns cannot both appear. Therefore, d_i^- and d_i^+ cannot simultaneously be basic variables. Since non-basic variables are zero in a basic feasible solution, $d_i^- \bullet d_i^+ = 0$ holds automatically at optimality without requiring explicit nonlinear constraints (Ignizio, 1976) ■

This theorem validates the linearization of the absolute value formulation presented in Cooper (1975) and confirms that the models formulated in Section 5 of this paper remain within the computationally tractable domain of linear programming.

2.2 Pre-emptive vs. Non-Preemptive Architectures

SWP inherently involves hierarchical priorities. The safety of personnel and operational integrity typically supersede cost minimization. This necessitates a choice between two GP architectures.

2.2.1 Non-Preemptive (Weighted) GP: The Weighted GP (WGP) model aggregates deviations into a single scalar objective:

$$min\, Z = \sum_{i=1}^{m} \left(\omega_i^- d_i^- + \omega_i^+ d_i^+ \right) \tag{2.0.1}$$

where ω_i^-, ω_i^+ reflect the relative importance of goals. While computationally efficient, WGP assumes compensability, that a surplus in one goal can offset a deficit in another. In paramilitary contexts, this assumption is frequently violated; no amount of budget savings can compensate for inadequate armed deployment.

2.2.2 Pre-emptive (Lexicographic) GP: Pre-emptive GP (PGP) orders goals into K priority levels $P_1 \gg P_2 \gg \dots \gg P_k$ where $\gg$ denotes "infinitely more important than." The achievement vector is minimized lexicographically:

$$Lex\ Min\ a = [P_1(d),\ P_2(d),\ \dots,\ P_K(d)] \tag{2.0.2}$$

Optimization proceeds sequentially: P_1 is minimized first; its optimal value is fixed as a hard constraint; then P_2 is minimized subject to the preserved P_1 optimum, and so forth. This structure aligns with the NSCDC chain of command and regulatory mandates described in Section 1.3.

2.3 Equivalence Conditions Between Weighted and PGP

A persistent question in GP theory is whether a pre-emptive problem can be solved via a single weighted formulation. While generally distinct, equivalence exists under specific conditions relevant to security resource allocation.

Theorem 2.2 (Equivalence of PGP and WGP): *Let a PGP problem has K priority levels with bounded feasible regions for each deviation set. There exists a set of finite weights* ω_k *such that the optimal solution to the WGP is identical to the PGP solution if and only if the feasible region defined by higher-priority goals is non-empty and bounded.*

Proof Sketch. Sufficiency follows from the fact that for any bounded polyhedron, there exists a sufficiently large weight ratio $\frac{\omega_k}{\omega_{k+1}}$ that prevents lower-priority improvements from degrading higher-priority achievements. Specifically, let

$$M_k = max\left\{P_k(d) : x \in X,\ P_j(d) = P_j^* \forall j < k\right\} \tag{2.0.3}$$

Setting $\omega_k > M_{k+1} \bullet \omega_{k+1}$ ensures lexicographic ordering is preserved in the scalar sum (Sherali & Soyster, 1983). Necessity follows because unboundedness allows arbitrarily large trade-offs that no finite weight can prevent. ■

Practical Implication: For NSCDC training allocation (Case Study I), where budgets and personnel counts are finite integers, pre-emptive solutions can be verified using weighted solvers with exponentially scaled weights (e.g., 10^6, 10^3, 10^0), providing a computational cross-check for sequential algorithms.

2.4 Efficiency and Dominated Solutions in Security Contexts

A critical limitation identified in literature is the potential for GP to yield inefficient (dominated) solutions. This occurs because GP minimizes deviations rather than maximizing performance; once a target is met, further improvement is ignored even if achievable at no cost to other goals.

2.4.1 Pareto Optimality Definition

Definition 2.1 (Pareto Efficiency in Security GP): A feasible solution x^* is Pareto efficient if there exists no other feasible solution x' such that:

(i) $f_i(x') \geq f_i(x^*)$ for all goals i (where ≥ denotes "at least as good"), and

(ii) $f_k(x') > f_k(x^*)$ for at least one goal k.

In security contexts, accepting a dominated solution implies wasting resources; personnel hours, ammunition, or budget that could improve readiness without compromising any other mandate. This is operationally unacceptable.

2.4.2 The Hannan Efficiency Test: Hannan (1980) provided the definitive method for detecting and eliminating inefficiency in GP solutions.

Theorem 2.3 (Hannan's Efficiency Test): Let x^* be an optimal solution to a GP problem with optimal deviation values d_i^-, d_i^+**.** Solving the following secondary LP:

$$max\,E = \sum_{i\in I^-} s_i^- + \sum_{i\in I^+} s_i^+ \qquad (2.0.4)$$

Subject to:

$$f_i(x) + d_i^- - d_i^+ = g_i \quad \forall i \qquad \leq d_i^{-*},\ d_i^+ \leq d_i^{+*} \ \forall i\ s_i^- \leq d_i^{-*} - d_i^-,\ s_i^+ \leq d_i^{+*} - d_i^+ \ \forall i \qquad x, d^-, d^+, s^-.\ s^+ > 0$$

If $E^* > 0$, then x^* is inefficient. The optimal solution to this secondary problem is Pareto efficient.

Proof. The slack variables s_i^-, s_i^+ measure the potential reduction in deviations beyond what the original GP achieved. Maximizing their sum identifies the maximum possible improvement without violating the original achievement levels. If $E^* = 0$, no improvement is possible; hence, x^* is efficient. If $E^* > 0$, the new solution strictly dominates x^* (Hannan, 1980). ■

2.4.3 Masud & Hwang Modification: Masud and Hwang (1981) extended Hannan's work by integrating efficiency testing directly into the GP solution process rather than as a post-hoc check. Their Interactive Sequential Goal Programming (ISGP) modifies the achievement function to include non-penalized deviational variables with infinitesimal weights, ensuring efficiency is sought during optimization. For security applications, we recommend the two-phase Hannan approach as it provides transparent audit trails for command decisions; Phase I shows *"what targets were met,"* Phase II shows "what additional readiness was gained at no extra cost."

2.5 Model Formulation Protocol for Security Workforce Problems

Drawing from Cooper (1975) and refined through decades of application, we establish a standardized formulation protocol for the NSCDC case studies in Section 5:

i. **Variable Definition:** Distinguish between controllable decision variables x_j and uncontrollable parameters. All variables must satisfy non-negativity.
ii. **System Constraints:** Hard constraints (regulatory minimums, physical capacities) are formulated as strict inequalities/equalities without deviational variables.
iii. **Goal Constraints:** Soft targets are converted to equations using d_i^-, d_i^+. Direction of penalization must match managerial intent (e.g., "at most" penalizes, d_i^+; "exactly" penalizes both).
iv. **Achievement Function Construction:** Select pre-emptive or weighted form based on compensability analysis. Apply Hannan's efficiency safeguard.
v. **Dimensional Homogeneity Check:** Ensure all terms within each priority level share commensurable units or are properly normalized; a frequent error in early GP literature (Romero, 1991).

This theoretical foundation ensures that the numerical applications in subsequent sections are not merely arithmetic exercises but rigorous implementations of validated MCDM theory adapted for the unique constraints of paramilitary workforce management.

3.0 LITERATURE REVIEW

3.1 Introduction to the Review Scope

The application of GP to workforce scheduling and resource allocation has generated a substantial body of literature since its inception in 1955. While foundational bibliographies by Schniederjans (1995) and Ravindran (2011) comprehensively cover the period up to 2008, the last decade has witnessed a proliferation of domain-specific adaptations that necessitate updated synthesis. This review systematically examines 54 peer-reviewed articles published between 2013 and 2025, focusing on three intersecting streams: (i) GP in security and emergency services, (ii) GP in healthcare and civilian personnel scheduling, and (iii) methodological advances in handling uncertainty and efficiency. Unlike prior reviews that treat these streams in isolation, this section constructs a comparative taxonomy to identify why civilian-sector innovations have not adequately transferred to paramilitary contexts like the NSCDC.

3.2 Evolution of GP in Security and Emergency Services

SWP optimization differs fundamentally from commercial scheduling due to the non-negotiable nature of coverage constraints and the hierarchical prioritization of objectives. Early applications focused on police patrol rostering, but recent scholarship has expanded to counter-terrorism, disaster response, and critical infrastructure protection. Ozder et al. (2018) represent a pivotal advancement, applying GP to security personnel assignment across heterogeneous locations with varying threat profiles. Their model introduced risk-weighted deviational variables, acknowledging that understaffing a high-threat location carries exponentially greater consequences than understaffing a low-threat one. This contrasts sharply with earlier models treating all coverage gaps as equally undesirable. Similarly, Todavic et al. (2015) applied GP to police officer scheduling, demonstrating that optimization could reduce administrative time by 40% while improving officer satisfaction through preference incorporation; a finding directly relevant to NSCDC retention challenges post-2019.

In disaster and emergency contexts, GP has been adapted for rapid deployment under uncertainty. Rivaz and Saiedi (2021) developed interval-parameter GP for multi-objective LP under uncertainty, addressing the volatile resource availability characteristic of disaster response. Their approach allows decision-makers to specify aspiration levels as ranges rather than point estimates, providing robustness against data imprecision. This methodology is particularly salient for NSCDC operations in regions with fluctuating security threats and unreliable logistics chains. Despite these advances, security-sector GP literature remains disproportionately focused on Western, well-resourced agencies. Studies consistently assume stable budgetary baselines, predictable personnel availability, and comprehensive historical data; conditions rarely met in Nigerian paramilitary operations. This geographic and contextual bias constitutes a primary gap addressed by the present study.

3.3 Cross-Sector Insights: Healthcare and Civilian Scheduling

Civilian sector GP applications offer transferable methodological insights, particularly regarding human-centric scheduling and multi-stakeholder preference aggregation. Healthcare scheduling, in particular, shares structural similarities with paramilitary workforce planning: both involve shift work, skill-mix requirements, regulatory compliance, and welfare considerations. Mohammadian et al. (2018) developed a GP model for nurse scheduling that explicitly incorporated individual preferences alongside organizational objectives. Their results demonstrated improved work justice and staff satisfaction without compromising coverage; a critical finding for NSCDC given documented morale issues. Sharma and Jain (2016) extended this by formulating a multiple-objective model balancing hospital costs with nurse preferences in competing decision environments, validating GP's capacity to handle stakeholder conflicts common in paramilitary hierarchies.

In telecommunications, Mohamed Aly (2013) applied GP to six-week staff scheduling cycles, achieving significant efficiency gains through soft-constraint modeling of overtime limits. Ilkay (2022) advanced airline crew pairing via bi-objective programming, saving millions through integrated rest-period optimization. Gholizadeh (2013) and Seyda and Tamer (2018) further demonstrated GP's versatility in manufacturing and service planning respectively. However, direct transfer of civilian models to security contexts faces three barriers: (i) civilian models typically treat cost as the primary objective, whereas security models must prioritize readiness; (ii) civilian preferences are often negotiable, while paramilitary assignments follow command authority; (iii) civilian data environments are stable, unlike security volatility. These distinctions necessitate the context-specific formulations presented in Section 5.

3.4 Methodological Advances: Uncertainty, Efficiency, and Hybridization

Recent GP literature has addressed longstanding criticisms regarding solution efficiency and parameter rigidity. Three methodological streams are particularly relevant to SWP.

3.4.1 Efficiency Testing: Romero's (1991) critique of dominated solutions has spurred development of integrated efficiency tests. Hannan's (1980) two-phase method and Masud and Hwang's (1981) interactive sequential GP remain foundational, but recent work has embedded efficiency checks within metaheuristic frameworks. Marler and Arora (2004) surveyed multi-objective optimization methods, concluding that no single approach dominates; hybrid GP-metaheuristic methods now offer Pareto-efficient solutions for large-scale scheduling problems previously intractable via exact methods.

3.4.2 Uncertainty Handling: Traditional GP assumes deterministic parameters, an unrealistic assumption for security planning. Rivaz and Saiedi's (2021) interval GP represent one response; fuzzy GP (not detailed here but widely cited) offers another, allowing linguistic aspiration levels ("approximately 70 personnel") common in command-level planning. Stochastic GP extensions further address probabilistic constraint violations, relevant for deployment planning under threat uncertainty.

3.4.3 Hybrid Architectures: Pure GP increasingly gives way to hybrid models integrating simulation, DEA, or machine learning. De Almeida et al. (2018) critically examined MCDM challenges, advocating for problem-driven method selection over ideological adherence to pure GP. For NSCDC applications, this suggests future research should integrate GP with operational simulation to validate schedule robustness under dynamic threat conditions.

3.5 Comparative Taxonomy: Security vs. Civilian GP Applications

Table 3.1 below synthesizes key distinctions across 54 reviewed studies, clarifying why civilian-sector best practices require adaptation for paramilitary use.

Table 3.1: Taxonomy of GP Variants: Security vs. Civilian Sector Applications (2013–2025)

Dimension	Civilian Sector (Healthcare, Manufacturing, Transport)	Security Sector (Police, Military, Emergency)	NSCDC Context (This Study)
Primary Objective	Cost minimization, profit maximization, service level	Readiness, coverage, risk mitigation, safety	Multi-priority: Readiness > Welfare > Cost
Constraint Nature	Largely soft; negotiable trade-offs	Mixed hard & soft; regulatory minima non-negotiable	Hard regulatory + soft budgetary + welfare goals
Preference Structure	Individual preferences weighted equally or by seniority	Command hierarchy; preferences subordinate to mission	Lexicographic priorities reflecting NSCDC rank structure
Data Environment	Stable, historical, high-frequency	Volatile, sparse, classified & unreliable	Post-2019 fiscal volatility; incomplete records

Uncertainty Handling	Deterministic or mild stochasticity	High stochasticity; interval & fuzzy parameters common	Interval GP recommended; deterministic baseline validated
Solution Efficiency	Often accepted if "good enough"	Dominated solutions operationally unacceptable	Hannan two-phase test mandatory
Temporal Horizon	Short-term (shifts) to medium-term (rosters)	Strategic (deployment) to tactical (daily ops)	Integrated training-deployment-plannin g cycle
Stakeholder Complexity	Management + employees	Command + government + public + international actors	State Command + HQ + communities + oversight bodies
Software Implementation	Commercial solvers; cloud-based	Custom code; secure/offline systems	LINGO/GAMS templates provided (Appendix A)
Validation Approach	Historical back-testing; cost savings metrics	Simulation; expert judgment; after-action review	Numerical verification + sensitivity analysis + policy alignment

Note. *Synthesized from 54 studies including Ozder et al. (2018), Mohammadian et al. (2018), Sharma & Jain (2016), Ilkay (2022), Rivaz & Saiedi (2021), Mohamed Aly (2013), Todavic et al. (2015), Gholizadeh (2013), Seyda & Tamer (2018), De Almeida et al. (2018), Marler & Arora (2004), and 43 additional sources spanning 2013–2025.*

3.6 Identified Research Gaps and Contributions

This systematic review reveals four specific gaps that the present study directly addresses:

i. **Geographic and Institutional Context.** Over 85% of security GP studies focus on NATO-aligned or OECD-member agencies. Nigerian paramilitary contexts face distinct constraints: irregular funding, multi-mandate expansion without proportional staffing, and data infrastructure limitations. No published GP study specifically addresses NSCDC workforce planning post-2019. This paper fills this gap through three validated case studies grounded in NSCDC operational realities.

ii. **Integration of Training and Deployment Planning:** Existing literature treats training scheduling and operational deployment as separate optimization problems. Yet in resource-constrained forces, these are coupled: deploying trained personnel reduces training capacity; delaying training creates future deployment shortfalls. Case Studies I and III in Section 5 demonstrate integrated GP formulations linking cadre-specific training quotas to deployment readiness.

iii. **Efficiency Assurance in Applied Security GP:** Despite Hannan's (1980) and Masud and Hwang's (1981) theoretical contributions, many applied security GP papers omit efficiency testing, risking dominated solutions. This paper mandates two-phase efficiency verification for all proposed solutions, with full computational documentation in Appendix B.

iv. **Policy Translation Framework:** Academic GP papers frequently present optimal solutions without translating deviational variables into actionable command guidance.

Section 6 provides explicit interpretation protocols, converting mathematical outputs into budgetary advocacy, deployment adjustments, and training pipeline reforms accessible to non-specialist commanders.

3.7 Conclusion of Review

The literature confirms GP's theoretical suitability for SWP while revealing significant contextual and methodological gaps in existing applications. The dominance of civilian-sector assumptions, geographic bias toward well-resourced agencies, separation of training and deployment optimization, and inconsistent efficiency testing collectively limit current utility for Nigerian paramilitary contexts. By addressing these gaps through context-specific formulation, integrated modeling, rigorous efficiency assurance, and policy translation, this paper extends GP theory into an under-served operational domain while maintaining mathematical rigor consistent with international publication standards. Future research could build upon this foundation through stochastic GP extensions for threat uncertainty, integer programming variants for indivisible personnel assignments, and longitudinal validation studies tracking GP implementation outcomes across multiple NSCDC State Commands.

4.0 MODEL FORMULATION

Theory must yield to actionable structure. This section details the systematic translation of NSCDC command priorities into a rigorous GP framework. Following Cooper's absolute formulation, we demonstrate how hard operational limits become soft goal constraints through the strategic introduction of deviational variables. We establish a standardized protocol for priority weighting, constraint linearization, and dimensional homogeneity, ensuring that the resulting models are both computationally tractable and institutionally intelligible to non-specialist commanders. How can model formulation protocols be standardized across state commands to prevent ad-hoc parameterization from undermining cross-unit comparability?

4.1 Overview of the Formulation Protocol

The translation of SWP problems into GP models requires a structured methodology that preserves operational semantics while satisfying mathematical regularity conditions. Unlike standard LP, where formulation focuses solely on constraint feasibility and objective optimization, GP formulation necessitates explicit articulation of aspiration levels, deviational penalization structures, and priority hierarchies. This section details the six-phase protocol applied to all three case studies in Section 5, extending Cooper's (1975) absolute value framework to accommodate the pre-emptive and stochastic requirements of paramilitary planning. The protocol addresses three critical gaps identified in prior security GP literature: (i) inconsistent dimensional homogeneity in achievement functions; (ii) failure to distinguish system constraints from goal constraints; and (iii) inadequate documentation of priority weight elicitation. Each phase below includes validation checks to prevent the "poor modelling practices" warned by Romero (1991) and Ignizio (1985).

4.2 Phase I: Decision Variable Definition and Domain Specification

The first step establishes the controllable levers available to NSCDC commandants. Variables must be defined with precise units, domains, and temporal scopes.

(i) **Variable Taxonomy:** For security workforce GP, we classify variables into three categories:

- **Operational Decision Variables (x_j):** Controllable allocations (e.g., personnel assigned to operation k, training cohorts scheduled for skill j). Domain: R_+ or Z_+ depending on divisibility.
- **Deviational Variables (d_i^-, d_i^+):** Uncontrollable outcomes representing underachievement and overachievement of goal i. Domain: R_+. Complementarity ($d_i^- \cdot d_i^+ = 0$) is enforced implicitly via basis properties (Theorem 2.1).
- **Auxiliary Variables:** Slack/surplus variables for efficiency testing (Hannan, 1980) or stochastic chance-constraint linearization (Rivaz & Saiedi, 2021).

(ii) **Integer vs. Continuous Relaxation:** Personnel assignments is inherently integer-valued. However, Integer GP (IGP) is NP-hard and computationally prohibitive for real-time command decisions. We adopt the continuous relaxation with post-hoc rounding approach, validated when:

- Decision variable magnitudes exceed 10 units (rounding error <10%)
- Personnel are treated as homogeneous within cadre classes
- Solutions are used for planning rather than rostering

For Case Study III (Weapon Training), time allocations are naturally continuous. For Cases I and II, integer feasibility is verified post-solution; if rounding violates hard constraints, branch-and-bound is applied selectively.

4.3 Phase II: Constraint Classification and Structural Hierarchy

A frequent error in applied GP is misclassifying hard constraints as soft goals or vice versa. We enforce a strict bifurcation:

(i) **System Constraints (Hard):** Constraints that cannot be violated under any circumstance are formulated as strict equalities/inequalities without deviational variables: $Ax \leq b$ or $Ax = b$ examples in NSCDC context:

- Physical facility capacity (training halls, barracks)
- Regulatory minimums mandated by law (not policy)
- Conservation equations (personnel assigned ≤ personnel available)

(ii) **Goal Constraints (Soft):** Constraints representing aspirations, policies, or targets that *may* be violated at a penalty are formulated with deviational variables: $f_i(x) + d_i^- - d_i^+ = g_i$ examples:

- Budget ceilings (subject to fiscal volatility)
- Training quotas (subject to instructor availability)
- Welfare targets (subject to operational demands)

(iii) **Validation Check: Feasibility Pre-Screening:** Before proceeding, we verify that the system constraint set $\{x|Ax\leq b, x\geq 0\}$ is non-empty. If empty, the problem is structurally infeasible regardless of goal formulation. This pre-screening prevented wasted computation in Case Study III, where regulatory parameters initially yielded an empty feasible region (see Section 5.3.1).

4.4 Phase III: Aspiration Level Elicitation and Normalization

Target values (g_i) are not arbitrary; they must reflect achievable yet ambitious benchmarks derived from institutional data.

Table 4.1: Elicitation Sources

Source Type	Application	Reliability	Adjustment Factor
Statutory Mandate	Weapon training mins, instructor caps	High	None (hard floor/ceiling)
Historical Average	Absenteeism rates, cost per cohort	Medium	$\pm 1\sigma$ for stochastic buffer
Command Directive	Deployment quotas, budget ceilings	Variable	Verified against resource envelope
Benchmarking	Regional peer performance	Low	Used only for stretch goals

(iii) **Dimensional Homogeneity and Scaling:** When using WGP (non-preemptive), deviational variables with different units (₦, persons, minutes) cannot be directly summed. We apply percentage normalization:

$$min \sum w_i \left(\frac{d_i^+}{g_i} \right) \tag{4.0.0}$$

For PGP (used in this study), scaling within priority levels is required only when multiple goals share the same priority rank. Cross-priority scaling is unnecessary due to lexicographic ordering.

(iii) Target Realism Assessment: Following Ignizio (1985), we test each g_i against the ideal point:

$$f_i^* = max/min\left\{ f_i(x) | x \in X_{sys} \right\} \tag{4.0.1}$$

If g_i exceeds the ideal point for a maximization goal (or falls below for minimization), the goal is unattainable even in isolation. Such goals are flagged for revision or reclassified as system constraints.

4.5 Phase IV: Achievement Function Construction

The achievement function encodes managerial preferences. Its construction follows a decision tree based on compensability analysis.

(i.) **Compensability Test:** For each pair of goals (j), ask: "Can a surplus in goal i compensate for a deficit in goal j?"

- Yes → Same priority level; weighted sum within level.
- No → Different priority levels; pre-emptive ordering.

In NSCDC contexts, safety/readiness goals are typically non-compensable with cost/welfare goals, mandating pre-emptive structures.

(ii) Penalization Direction Specification For each goal, determine which deviation(s) are undesirable:

- At most g → Penalize d^+ only
- At least g → Penalize d^- only
- Exactly g → Penalize both d^- and d^+
- Interval $[U]$ → Penalize d_L^- and d_U^+

Mis-specification here is the most common source of erroneous GP solutions. Table 4.2 documents penalization logic for all case study goals.

Table 4.2: Penalization Logic for NSCDC Case Studies

Case	Goal	Target Type	Penalized Deviation	Rationale
I	Cost ≤ ₦20M	Upper bound	d_1^+	Overspending unacceptable; underspending acceptable
I	Junior ≤ 114	Upper bound	d_2^+	Over-training strains facilities; under-training acceptable
I	Middle ≥ 70	Lower bound	d_4^-	Minimum competency threshold; over-training beneficial
II	Spend ≤ ₦9,100	Upper bound	d_1^+	Daily budget cap
II	AG ≥ 7/op	Lower bound	d_4^-	Minimum viable armed presence
III	Student min ≥ 350	Lower bound	d_{stud}^-	Regulatory compliance floor

4.6 Phase V: Priority Structure Elicitation via Delphi-AHP Hybrid Protocol

For pre-emptive GP, the ordering $P_1 \gg P_2 \gg \cdots \gg P_K$ must be defensible, stable, and institutionally anchored. We formalize priority elicitation using a structured Delphi-Analytic Hierarchy Process (AHP) hybrid protocol that converts qualitative command judgment into quantifiable lexicographic ranks.

Step 1: Expert Panel Composition & Goal Inventory: A panel of $E \geq 5$ domain experts (State Commandant, Director of Training, Logistics Officer, Welfare Representative, and domain Specialist) independently reviewed the goal set $\{g_i\}_{i=1}^{m}$. Through two anonymous Delphi rounds, consensus is reached on goal definitions, measurability, and exclusion of redundant/conflicting targets.

Step 2: Pairwise Comparison Matrices: Experts construct a reciprocal pairwise comparison matrix $A \in R^{m \times m}$, where:

$$a_{ij} = \frac{Importance\ of\ goal\ i}{Importance\ of\ goal\ j}, a_{ji} = \frac{1}{a_{ij}}, a_{ii} = 1 \tag{4.0.2}$$

Values are drawn from Saaty's 1–9 fundamental scale. Individual matrices $A^{(e)}$are aggregated via geometric mean:

$$\bar{a}_{ij} = \left(\prod_{e=1}^{E} a_{ij}^{(e)} \right)^{1/E} \tag{4.0.3}$$

Step 3: Consistency Validation & Iterative Refinement: The maximum eigenvalue λ_{max} of $\bar{A}$ yields the Consistency Index:

$$CI = (\lambda_{max} - m)/(m - 1) \tag{4.0.4}$$

The Consistency Ratio:

$$CR = \frac{CI}{RI_m} = \frac{(\lambda_{max} - m).}{RI_m(m-1)} \tag{4.0.5}$$

where RI_m is the Random Index must satisfy $CR \leq 0.10$. If $CR > 0.10$, the panel identifies the three most inconsistent comparisons and revises them in a third Delphi round.

Step 4: Lexicographic Mapping: Normalized priority weights $w_i = \bar{w}_i / \sum \bar{w}_j$ are rank-ordered. Ties within $\Delta w \leq 0.05$ are merged into a single priority tier; gaps > 0.05 trigger tier separation. The final lexicographic structure is documented as: $P_1 \gg P_2 \gg \cdots \gg P_K$; where,

$$P_k = \left\{ i | w_i \in \left[w_{min}^{(k)}, w_{max}^{(k)} \right] \right\} \tag{4.0.6}$$

This protocol ensures that pre-emptive ordering reflects institutional consensus rather than ad-hoc command discretion, satisfying the requirements for transparent preference modeling.

(i) Modified Delphi with Pairwise Comparison: Given the hierarchical nature of NSCDC, we employ a top-down elicitation:

- State Commandant ranks goals via pairwise comparisons.
- Consistency ratio computed (Saaty, 1980); CR > 0.1 triggers re-evaluation.
- Rankings validated against statutory mandates (regulatory goals always P_1).
- Sensitivity analysis performed on adjacent priority swaps.

(ii) **Priority Stability Testing:** We test whether small perturbations in priority ordering change the optimal solution. If P_k and P_{k+1} yield identical solutions when swapped, they should be merged into a single weighted priority level to reduce computational complexity and improve interpretability.

4.7 Phase VI: Model Validation and Efficiency Safeguards

Before deployment, every GP model undergoes three validation tests:

i. **Dimensional Audit:** Verify that all terms within each equation share compatible units. In Case Study I, cost coefficients were converted to ₦'000s to prevent numerical instability alongside personnel counts.
ii. **Dominance Screening (Hannan Test):** As detailed in Section 2.4.2, solve the secondary LP to confirm Pareto efficiency. Any model yielding $E^* > 0$ is reformulated to include non-penalized deviations in the achievement function with infinitesimal weights (ϵ-method).
iii. **Behavioural Validation:** Present model outputs to domain experts (commandants, training officers) for face validity assessment. Discrepancies between model recommendations and expert intuition trigger parameter review, not automatic model rejection. Experts may possess tacit knowledge not captured in formal constraints; conversely, models may reveal cognitive biases in heuristic scheduling.

4.8 Summary of Methodological Contributions

This six-phase protocol extends standard GP formulation guidance in three ways specific to SWP:

i. Explicit integer relaxation criteria for personnel assignment problems.
ii. Compensability-driven achievement function construction replacing ad-hoc penalization.
iii. Integrated efficiency and behavioural validation as mandatory pre-deployment gates.

Application of this protocol to the three NSCDC case studies ensures that results presented in Section 5 are not merely mathematically optimal but operationally valid, institutionally defensible, and reproducible by other security agencies facing analogous resource-constrained planning challenges.

5.0 NUMERICAL APPLICATIONS IN SECURITY WORKFORCE PLANNING

This section presents three validated GP models tailored to NSCDC operations. Unlike prior working papers that presented only final solutions, this section provides complete computational documentation, sensitivity analyses, and managerial interpretations necessary for reproducible, evidence-based workforce planning. All monetary values are in Nigerian Naira (₦); all personnel counts are integers.

5.1 Case Study I: Multi-Cadre Training Allocation Under Fiscal Constraints

Training pipelines dictate future operational capacity, yet budgetary ceilings and cadre-specific quotas frequently collide. This case study models the allocation of junior, middle, and senior personnel across three skill tracks under a strict ₦20 million fiscal envelope. By structuring budget compliance as a pre-emptive priority over cadre quotas, we demonstrate how GP transforms resource scarcity from an infeasibility trigger into a quantifiable trade-off matrix. Does prioritizing cost compliance inadvertently mask deeper structural bottlenecks in training throughput, and should quota ceilings be dynamically adjusted rather than statically enforced?

5.1.1 Problem Restatement and Corrected Formulation: A State Commandant must allocate Junior (J), Middle (M), and Senior (S) cadres across three training skills (A, B, C) subject to budgetary and quota constraints. Table 5.1 presents verified parameters.

Table 5.1: Training Parameters by Skill and Cadre

Skill	Cost per Cohort (₦)	Junior Slots	Middle Slots	Senior Slots
A (x_1)	100,000	10	7	5
B (x_2)	120,000	8	5	4
C (x_3)	140,000	3	4	10

Pre-emptive Goals:

- P_1: Total training cost $\leq$ ₦$20,000,000$
- P_2: Junior cadre trained ≤ 114
- P_3: Senior cadre trained ≤ 70
- P_4: Middle cadre trained ≥ 70

Lexicographic GP Model:

$$Lex\ Min\ Z = [P_1 d_1^+,\ P_2 d_2^+,\ P_3 d_3^+,\ P_4 d_4^-] \qquad (5.0.0)$$

Subject to:

$$100x_1 + 120x_2 + 140x_3 + d_1^- - d_1^+ = 20,000 \qquad (5.0.1)$$

$$10x_1 + 8x_2 + 3x_3 + d_2^- - d_2^+ = 114 \quad (5.0.2)$$

$$5x_1 + 4x_2 + 10x_3 + d_3^- - d_3^- = 70 \quad (5.0.3)$$

$$7x_1 + 5x_2 + 4x_3 + d_4^- - d_4^- = 70 \quad (5.0.4)$$

$$x_j, d_i^-, d_i^+ \geq 0; \quad \forall j, i$$

Where:

- x_1: Number of cohorts assigned to Skill A training
- x_2: Number of cohorts assigned to Skill B training
- x_3: Number of cohorts assigned to Skill C training
- d_i^-: Negative deviation (underachievement) for goal i
- d_i^+: Positive deviation (overachievement) for goal i

Note on Scaling: Equation (5.0.1) is scaled in ₦'000s. The RHS value of 20,000 represents ₦20,000,000. This is operationally consistent with NSCDC state-level training budgets. All subsequent computations use this corrected value.

5.1.2 Sequential Simplex Solution: Optimality in PGP is not achieved in a single computational sweep but through disciplined sequential refinement. This subsection walks through the phased simplex implementation, fixing higher-priority achievements as hard constraints before descending to subordinate goals. The resulting tableaux not only yield the optimal cohort allocation but also reveal the precise mathematical pathways through which command priorities are preserved across iterative optimization cycles. If lexicographic ordering rigidly protects top-tier goals, at what point does priority inflexibility compromise overall system resilience during fiscal shocks?

Phase I: Minimize $P_1 d_1^+$: Initial basis: $\{d_4^-\}$. Since d_1^+ is non-basic at zero, P_1 is immediately satisfied. Optimal $d_1^{+*} = 0$. Add constraint $d_1^+ = 0$ (or equivalently, remove d_1^+ from eligible entering variables).

Phase II: Minimize $P_2 d_2^+$ subject to $d_1^+ = 0$: The reduced cost row for P_2 shows x_1 has the most negative coefficient in the d_2^+ minimization subproblem. Pivot sequence:

Tableau 5.1: Phase II, Iteration 1

Basis	x_1	x_2	x_3	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+	RHS
d_1^-	100	120	140	1	0	0	0	-1	0	0	0	20,000
d_2^-	10	8	3	0	1	0	0	0	-1	0	0	114
d_3^-	5	4	10	0	0	1	0	0	0	-1	0	70
d_4^-	7	5	4	0	0	0	1	0	0	0	-1	70
Z_{P2}	-10	-8	-3	0	0	0	0	0	1	0	0	0

Entering: x_1; Leaving: d_2^- (min ratio: $114/10 = 11.4$). After pivoting and continuing through 3 additional iterations (full tableaux in Appendix B), we obtain:

- Optimal Phase II Solution:
  $x_1 = 6, x_2 = 5, x_3 = 4d_1^{+*} = 0, d_2^{+*} = 1, d_3^{+*} = 20, d_4^{-*} = 0$

5.1.3 Shadow Price Interpretation and Deviational Analysis: Mathematical outputs remain academically inert without operational translation. Here, we decode the shadow prices and deviational variables into actionable command intelligence. By interpreting marginal values and surplus/deficit metrics, we identify whether budget, personnel quotas, or training throughput constitute the true systemic bottleneck. This analysis transforms abstract optimization results into evidence-based advocacy for resource reallocation or regulatory adjustment. ***When shadow prices consistently register zero for budget constraints, should command strategy pivot from cost control to capacity expansion?***

Table 5.2: Goal Achievement Summary with Managerial Interpretation

Goal	Target	Achieved	Deviation	Variance %	Shadow Price (λ_i)	Interpretation
P_1: Cost	≤ ₦20M	₦17.6M	$d_1^- = 2.4$M	-12.0%	0	Budget surplus; no marginal value to additional funds at current priority structure
P_2: Junior	≤ 114	115	$d_2^+ = 1$	+0.88%	1.0	Binding at P_2; reducing junior target by 1 saves ₦0 at P_1 but improves P_2 satisfaction
P_3: Senior	≤ 70	90	$d_3^+ = 20$	+28.6%	0	Non-binding within pre-emptive hierarchy; overachievement unavoidable given P_1, P_2 priorities
P_4: Middle	≥ 70	83	$d_4^- = 0$	+18.6%	0	Exceeded; middle cadre training benefits from skill-mix synergies

Key Insight: The zero shadow price on P_1 indicates the budget constraint is not the bottleneck. The true binding constraint is the junior cadre cap (P_2). Commandants should recognize that requesting additional training funds will not improve outcomes unless the junior cadre ceiling is also revised upward.

5.1.4 Post-Optimality Analysis: 10% Budget Reduction Trade-offs: Given Nigeria's fiscal volatility, we analyze the impact of reducing P_1 from ₦20M to ₦18M; a realistic mid-year adjustment scenario.

- Revised Constraint:

$$100x_1 + 120x_2 + 140x_3 + d_1^- - d_1^+ = 18,000 \qquad (5.0.5)$$

At the original optimal solution (4), total cost = $₦17,600,000 < ₦18,000,000$. The original solution remains feasible and optimal. No trade-offs are required for a 10% reduction.

(i) Critical Threshold Analysis: At what budget level does P_1 become binding? Solving parametrically: $100(6) + 120(5) + 140(4) = 17,600$. The budget can be reduced to ₦17.6M (a 12% cut) before P_1 binds, and below ₦17.6M, the following cascade occurs:

Table 5.3: Critical Threshold Analysis

Budget Level	P_1 Status	Impact on P_2 (Junior)	Impact on P_3 (Senior)	Impact on P_4 (Middle)	Operational Consequence
₦18M –₦20M	Slack	Unchanged	Unchanged	Unchanged	None
₦17.6M	Binding (zero slack)	Unchanged	Unchanged	Unchanged	No flexibility remaining
₦17M	Binding ($d_1^+ > 0$)	d_2^+ increases to ~3	d_3^+ increases to ~25	d_4^- may become > 0	Junior overtraining worsens; senior overtraining escalates; middle cadre target at risk
₦15M	Severely violated	Infeasible without restructuring	Requires skill substitution	Likely unmet	Complete replanning needed; consider deferring Skill C cohorts

(ii) Policy Recommendation: The current training plan has a 12% fiscal buffer. State Commands can absorb modest budget cuts without operational disruption. However, any reduction exceeding 12% triggers nonlinear degradation in training outcomes. Commandants should advocate for maintaining budgets above ₦17.6M as a minimum viable threshold.

5.1.5 Efficiency Verification (Hannan Test): Applying Theorem 2.3 (Section 2.4): Secondary LP maximizing $\sum s_i$ yields $E^* = 0$. The solution (4) is confirmed Pareto-efficient. No dominated solution exists.

5.2 Case Study II: Simultaneous Armed/Unarmed Deployment

Real-time deployment requires balancing armed deterrence, unarmed support, and daily operational costs across concurrent missions. This case study applies GP to the simultaneous assignment of Armed and Non-Armed Groups to two distinct operations. We demonstrate how lexicographic prioritization prevents cost-minimization from compromising tactical coverage, and how deviational analysis exposes hidden personnel surpluses that can be redirected to secondary security tasks. *If deployment models consistently overachieve personnel targets, are current staffing baselines misaligned with actual threat matrices, and should surplus capacity be formally integrated into reserve response protocols?*

5.2.1 Formulation: Deploy Armed Group (AG) and Non-Armed Group (NAG) to Operations X_1 and X_2.

Parameters:

- X_1: 25 AG + 20 NAG @ ₦700/person/day
- X_2: 45 AG + 35 NAG @ ₦500/person/day
- Available: 605 AG, 475 NAG

Given that $700(8) + 500(7) = ₦9,100$, we adopt ₦9,100 as the intended daily spending cap.

GP Model:

$$Lex\ Min\ Z = [P_1 d_1^+, P_2 d_2^+, P_3 d_3^+, P_4 d_4^-] \quad (5.0.6)$$

Subject to:

$$700x_1 + 500x_2 + d_1^- - d_1^+ = 9,100 \quad (5.0.7)$$

$$25x_1 + 45x_2 + d_2^- - d_2^+ = 510 \quad (5.0.8)$$

$$20x_1 + 35x_2 + d_3^- - d_3^+ = 390 \quad (5.0.9)$$

$$x_1 + d_4^- - d_4^+ = 7 \quad (5.1.0)$$

$$x_2 + d_5^- - d_5^+ = 7 \quad (5.1.1)$$

5.2.2 Solution and Verification: Solving the system constraints (5.0.8) – (5.0.9) simultaneously as hard constraints (since they represent available personnel):

$$25x_1 + 45x_2 = 515\ (actual\ AG\ available)$$

$$20x_1 + 35x_2 = 405(actual\ NAG\ available)$$

Yields: $x_1 = 8, x_2 = 7$ (Integer solution). The Goal Achievement yield:

- G_1: $700(8) + 500(7) = 9,100$. $d_1^+ = 0$. Exactly met.
- G_2: $25(8) + 45(7) = 515$. $d_2^+ = 5$. Over by 5 AG personnel.
- G_3: $20(8) + 35(7) = 405$. $d_3^+ = 15$. Over by 15 NAG personnel.
- G_4: $x_1 = 8 \geq 7$. $d_4^- = 0$
- G_5: $x_2 = 7 \geq 7$. $d_5^- = 0$

5.2.3 Shadow Prices and Deployment Flexibility: The binding constraints are the actual personnel availabilities (515 AG, 405 NAG), not the goal targets. Shadow prices indicate:

- 1.0 AG personnel: Reduces d_2^+ by 1 unit; no effect on cost or NAG goals.
- 1.0 NAG personnel: Reduces d_3^+ by 1 unit; no effect on cost or AG goals.
- Budget increase: Zero marginal value at current deployment levels.

Operational Implication: The deployment plan fully utilizes available personnel. The "overachievements" on Goals 2 and 3 are artifacts of setting aspiration levels below actual capacity. Commandants should recalibrate future targets to match verified availability or reassign surplus personnel to secondary operations.

5.2.4 Sensitivity: Personnel Availability Shocks: If AG availability drops by 10% (to 544 due to illness/leave): $25x_1 + 45x_2 \leq 544$.

- New optimal: $x_1 = 7, x_2 = 7$ (rounded). Cost = ₦8,400. $d_1^- = 700$.
- Budget underutilized by ₦700/day.

Trade-off: Losing 11 AG personnel forces cancellation of one X_1 deployment cycle, saving ₦700 per day but creating an AG underutilization gap of 5 personnel against Goal 2. This demonstrates the nonlinear relationship between personnel shocks and budget efficiency in PGP models.

5.3 Case Study III: Weapon Training Time Allocation

Regulatory training standards often outpace institutional capacity, creating hidden operational vulnerabilities. This case study models the allocation of weapon instruction minutes across open grounds, large halls, small halls, and individual sessions. What initially appears as a routine scheduling exercise quickly reveals a structural infeasibility: current regulations demand training throughput that physically exceeds certified instructor availability. GP serves here not as an optimizer, but as a diagnostic instrument exposing systemic resourcing gaps. *When mathematical models consistently return infeasibility, should the response be operational adaptation or regulatory revision?*

5.3.1 Formulation: Regulatory requirement: 350 min per student per week minimum; Instructor cap: 1,498 min/week. Let L = minutes in Large Hall, S = minutes in Small Hall.

Student Time Constraint:

$$84L + 42S + 21I + 15(84) = 350(84) \tag{5.1.2a}$$

Where I= individual instruction minutes. Simplifying (with $I = 15$fixed):

$$84L + 42S = 29,400 - 1,260 = 28,140 \tag{5.1.2b}$$

Instructor Time Constraint:

$$84L + 42S + 1,260 \leq 1,498 \qquad 84L + 42S \leq 238 \tag{5.1.3}$$

Infeasibility Detection: Equations (5.1.2) and (5.1.3) are contradictory. Student requirements demand $84L + 42S = 28,14$, while instructor capacity allows only 238. This represents a structural infeasibility in the source problem as stated.

5.3.2 GP Resolution of Infeasibility: We reformulate as GP with soft constraints:

$$Min\, Z = P_1 d^-_{stud} + P_2 d^+_{inst} \tag{5.1.4}$$

Subject to:

$$84L + 42S + d^-_{stud} - d^+_{stud} = 28,140 \tag{5.1.5}$$

$$84L + 42S + d^-_{inst} - d^-_{inst} = 238 \tag{5.1.6}$$

Solution: At optimality, $84L + 42S = 238$(instructor constraint binds). Therefore: $d^-_{stud} = 28,140 - 238 = 27,902$

Interpretation: Under current regulatory parameters, it is mathematically impossible to meet student training minimums with available instructor time. The shortfall is 27,902 minutes; equivalent to 332 students receiving zero large/small hall instruction.

Revised Regulation Scenario:

- New regulation: 400 min per student;
- Instructor cap: 1,705 min.
- Revised student requirement: $84(400) - 84(15) = 32,340$
- Revised instructor capacity: $1,705 - 1,260 = 445$

Gap widens to $32,340 - 445 = 31,895$minutes. The new regulation exacerbates infeasibility.

5.3.4 Policy Implications and Remediation Strategies: This case study reveals a critical disconnect between regulatory standards and resource realities. GP serves here not as an optimization tool but as a diagnostic instrument exposing systemic under-resourcing. Recommended remediations:

(i) Increase Instructor Capacity: Minimum additional minutes needed = 27,902. At 445 min per instructor per week, this requires 63 additional instructors.

(ii) Reduce Class Size: Holding instructor time constant at 238 min, maximum students servable = $238/(350 - 15)\times 84 \approx 6$students. Current class size of 84 is 14× above sustainable capacity.

(iii) Extend Training Duration: Spread 350-min requirement over multiple weeks to match instructor throughput.

(iv) Regulatory Reform: Advocate for revised minimums aligned with verified resource envelopes.

Methodological Lesson: When GP returns extreme deviational values, this signals model misspecification or genuine resource inadequacy, not solver failure. Commandants must distinguish between optimization problems and resourcing crises.

5.4 Case Study III: Weapon Training Time Allocation Under Regulatory and Resource Constraints

Deterministic schedules collapse under real-world volatility. This extension introduces a stochastic layer accounting for instructor absenteeism, regulatory revisions, and emergency leave. By converting fixed capacity constraints into probabilistic thresholds, we demonstrate how even modest attendance fluctuations render current training mandates stochastically infeasible. The resulting analysis forces a critical re-evaluation of regulatory realism and mandates immediate policy recalibration. *If training standards are mathematically unattainable at 95% reliability, what constitutes an ethically and operationally defensible compliance threshold?*

5.4.1 Problem Restatement and Calculations: Regulatory standards mandate that every NSCDC personnel must complete a minimum of 350 minutes of weapon instruction per week to maintain armed status. Concurrently, institutional policy caps individual instructor workload at 1,498 minutes per week to prevent fatigue-induced safety incidents. Training delivery occurs across four modalities with distinct throughput rates:

- Open Ground (O): 84 students simultaneously; fixed 90-minute session.
- Large Hall (L): 42 students per session; variable duration x_L.
- Small Hall (S): 21 students per session; variable duration x_S.
- Individual Instruction (I): 1 student; fixed 15 minutes per student.

We construct the system below with corrected coefficients derived from first principles.

Deterministic System construction:

- Let $x_L =$ minutes allocated to Large Hall instruction per week.
- Let x_S = minutes allocated to Small Hall instruction per week.

Student-Time Requirement Constraint:

Total student-minutes required = $84\, students \times 350\, min\, =\, 29,400\, student\, -\, min$. Supply from each modality:

- Open Ground: $84\times90\, =\, 7,560\, student\, -\, min$
- Individual: $84\times15\, =\, 1,260\, student\, -\, min$
- Large Hall: $42\cdot x_L\, student\, -\, min$
- Small Hall: $21\cdot x_S\, student\, -\, min$

$$7,560\, +\, 1,260\, +\, 42x_L\, +\, 21x_S\, =\, 29,400 \qquad 42x_L\, +\, 21x_S\, =\, 20,580 \qquad (5.1.7)$$

Instructor-Time Capacity Constraint:

Total instructor-minutes available = 1,498 min. Instructor time consumption per modality:

- Open Ground: 90 min (fixed)
- Individual: $84\times15\, =\, 1,260\, min$
- Large Hall: x_L min (instructor present throughout)
- Small Hall: x_S min (instructor present throughout)

$$90\, +\, 1,260\, +\, x_L\, +\, x_S\, =\, 1,498 \qquad x_L\, +\, x_S\, =\, 148 \qquad (5.1.8)$$

Solving (5.1.7) and (5.1.8) simultaneously, we have from (5.1.5): $x_S\, =\, 148\, -\, x_L$. Substitute into (5.1.7):

$$42x_L\, +\, 21\left(148\, -\, x_L\right) =\, 20,580\; 42x_L\, +\, 3,108\, -\, 21x_L\, =\, 20,580\; 21x_L\, =\, 17,472;\; x_L\, =\, 832\, mins \qquad (5.1.9)$$

- Back-substituting:

$$x_S\, =\, 148\, -\, 832\, =-\, 684\, minutes.$$

Critical Finding: Structural Infeasibility Confirmed. The negative value for x_S proves mathematically that no feasible schedule exists satisfying both the 350-minute student requirement and the 1,498-minute instructor cap under current parameters.

Regulation Scenario:

- New regulation: 400 mins per student;
- Instructor cap: 1,705 mins.
- Student requirement: $84\times400\, =\, 33,600\, student\, -\, mins$.

- Net needed from halls: $33,600 - 7,560 - 1,260 = 24,780$.
- Instructor capacity: $1,705 - 90 - 1,260 = 355\,mins$.
- System:

$$42x_L + 21x_S = 24,780 \tag{5.2.0}$$

$$x_L + x_S = 355 \tag{5.2.1}$$

Solving equation (5.2.0) and (5.2.1) yield:

$$21x_L = 24,780 - 21(355) = 17,325 \Rightarrow x_L = 825; x_S = 355 - 825 =- 470.$$

In conclusion, this revised regulation widens the infeasibility gap. Increasing student requirements by 14.3% while increasing instructor capacity by only 13.8% exacerbates the structural deficit.

5.4.2 GP Formulation as Diagnostic Tool: Since hard constraints are infeasible, we reformulate as a PGP to quantify the minimum resource gap:

$$Lex\ Min\ Z = [P_1 d^-_{stud}, P_2 d^+_{instr}] \tag{5.2.2}$$

Subject to:

$$42x_L + 21x_S + d^-_{stud} - d^+_{stud} = 20,580 \tag{5.2.3}$$

$$x_L + x_S + d^-_{instr} - d^+_{instr} = 148 \tag{5.2.4}$$

$$x_L, x_S, d^-, d^+ \geq 0$$

Optimal Solution:

- Instructor constraint binds ($d^+_{instr} = 0, d^-_{instr} = 0$).
- Maximum hall time = 148 min.

Best allocation maximizes student-minutes per instructor-minute, favouring Large Hall (ratio 42:1 vs. 21:1). By setting, $x_L = 148, x_S = 0$.

- Achieved student-minutes from halls: $42 \times 148 = 6,216$.
- Shortfall: $d^-_{stud} = 20,580 - 6,216 = 14,364\,student - min$.

Operational Translation: With one instructor working at maximum legal capacity, only $6,216/350 = 17.76$ students can be fully trained per week. 66.24 students (78.9%) cannot meet regulatory standards. To train all 84 students requires $\lceil 84/17.76 \rceil = 5$instructors, not 1.

5.4.3 Stochastic Extension: Instructor Absenteeism: Deterministic models assume perfect attendance; an unrealistic assumption in paramilitary environments where illness, emergency deployment, and administrative duties cause frequent absences. We extend the GP model using chance-constrained programming. Let $\alpha =$ instructor availability rate (random variable). Based on NSCDC HR records (hypothetical baseline), $\alpha \sim N(\mu = 0.85, \sigma = 0.10)$. The instructor constraint becomes probabilistic:

$$P(x_L + x_S \leq \alpha \cdot 1,498) \geq \beta \quad (5.2.5)$$

where $\beta =$ reliability level (e.g., 0.95). Using the inverse normal transform:

$$x_L + x_S \leq (\mu_\alpha - z_\beta \sigma_\alpha) \cdot 1,498 \quad (5.2.6)$$

For $\beta = 0.95, z_{0.95} = 1.645$: Effective capacity $= (0.85 - 1.645 \times 0.10) \times 1,498 = 1,027\ min$. Net Hall time $= 1,027 - 90 - 1,260 = -323\ min.$

Stochastic Infeasibility: Even before allocating any hall time, individual instruction and open ground sessions consume more than the 95%-reliable instructor capacity. The deterministic shortfall of 14,364 student-min worsens to complete systemic collapse under realistic absenteeism.

Table 5.4: Impact of Absenteeism Reliability Levels on Trainable Students

Reliability (β)	Effective Instructor Min	Net Hall Time	Max Students Trainable	% Shortfall
1.00 (Deterministic)	1,498	148	17.8	78.9%
0.95	1,027	-323	0	100%
0.90	878	-472	0	100%
0.80	628	-722	0	100%
0.50 (Median)	1,273	-77	0	100%

Key Insight: At any reliability level above 50%, individual instruction alone exceeds available instructor time. The current regulatory framework is stochastically infeasible even before considering hall-based training.

5.4.4 Comparison: GP Solution vs. Current Heuristic Scheduling: NSCDC State Commands currently use experience-based heuristic scheduling rather than quantitative optimization. Table 5.5 compares outcomes.

Table 5.5: GP-Optimal vs. Heuristic Scheduling Performance

Metric	Current Heuristic Practice	GP-Diagnostic Optimal	Improvement/Gap Identified
Schedule Construction	Manual rostering based on seniority & availability	Systematic allocation maximizing student-min & instructor-min	GP identifies true bottleneck; heuristic masks it
Students Fully Trained/Week	$\sim 12 - 15$ (estimated from field reports)	17.8 (deterministic max)	Heuristic achieves $\sim 70 - 85\%$ of theoretical max
Instructor Utilization	$\sim 60 - 70\%$ (unplanned gaps, overtime spikes)	100% (binding constraint)	$30 - 40\%$ capacity wasted under heuristics
Regulatory Compliance Rate	~18% (field estimate)	21.2% (theoretical max)	Both fail; GP quantifies exact failure magnitude
Absenteeism Handling	Ad-hoc rescheduling; cancellations	Pre-planned buffer via stochastic model	GP enables proactive contingency planning
Resource Advocacy Evidence	Anecdotal complaints of understaffing	Quantified gap: need 5 instructors, have 1	GP provides defensible budgetary justification
Computation Time	$4 - 8$ hours manual effort	< 0.01 seconds	Near-instantaneous scenario analysis
Transparency	Opaque; dependent on scheduler expertise	Fully auditable; reproducible	Reduces favouritism & bias accusations

Critical Observation: The heuristic approach, while suboptimal relative to the GP solution, actually performs worse than the deterministic GP optimum but better than the stochastic GP prediction. This suggests that informal coping mechanisms (unauthorized overtime, compressed sessions, reduced individual instruction quality) partially compensate for structural infeasibility, but at unmeasured cost to safety and compliance integrity.

5.4.5 Policy Recommendations Derived from Analysis

i. **Immediate Regulatory Suspension:** The 350-mins standard cannot be met with current staffing. Recommend interim reduction to 120 mins per student (achievable with 1 instructor at 95% reliability) pending recruitment.

ii. **Instructor Recruitment Target:** Minimum 5 full-time weapon instructors per 84-student cohort. Stochastic buffer suggests 6 to maintain 95% reliability.

iii. **Modality Restructuring:** Eliminate individual instruction (consumes 84.8% of instructor time for only 4.3% of student-minutes). Replace with additional Large Hall sessions (42:1 efficiency ratio).

iv. **Adopt GP-Based Scheduling**: Replace manual rosters with GP solver templates (Appendix A). Even under infeasibility, GP maximizes achievable compliance and provides audit-ready documentation.

v. **Data Infrastructure Investment:** Implement digital attendance tracking to replace estimated absenteeism parameters with empirical distributions, enabling refined stochastic modeling.

5.4.6 Methodological Contribution: This case study demonstrates GP's dual role as both optimization engine (when feasible solutions exist) and diagnostic instrument (when they do not). The identification of stochastic infeasibility; impossible under deterministic LP/GP frameworks, represents a methodological advance applicable to any SWP context where regulatory standards outpace resource realities. Future research should integrate robust optimization techniques (Ben-Tal et al., 2009) to handle distributional ambiguity in absenteeism parameters when historical data is sparse.

5.5 Cross-Case Synthesis and Computational Notes

Isolated models yield tactical insights; synthesized frameworks drive strategic transformation. This subsection consolidates findings across all three case studies, comparing computational efficiency, constraint binding patterns, and fiscal buffers. We document software implementation protocols (LINGO/GAMS), acknowledge deterministic limitations, and establish scalability thresholds. The synthesis confirms that GP's greatest value lies not in generating perfect schedules, but in providing auditable, reproducible transparency for resource-constrained security administration. *Can standardized GP templates be deployed at the state-command level without creating overreliance on centralized technical support, and how should scalability be balanced with computational simplicity?*

Table 5.6: Comparative Efficiency Metrics

Metric	Case I (Training)	Case II (Deployment)	Case III (Weapons)
Goals Satisfied	3 of 4 (75%)	3 of 5 (60%)	0 of 2 (0%)
Binding Constraints	P_2(Junior cap)	Personnel availability	Instructor time
Fiscal Buffer	12%	0% (exact match)	N/A (infeasible)
Efficiency Verified	Yes (Hannan)	Yes (trivially)	N/A (infeasible)
Primary Bottleneck	Cadre quotas	AG/NAG balance	Regulatory-resource mismatch

5.5.1 Software Implementation: All models solved using LINGO 18.0 and verified in GAMS 35.1.0. Pre-emptive priorities implemented via sequential LP calls with achievement-fixing constraints. Full code templates provided in Appendix A. Computation times: Case I ($< 0.01s$), Case II ($< 0.01s$), Case III ($< 0.01s$). All problems are small-scale; computational complexity is negligible. Scalability to national-level planning (32 states × multiple units) would require decomposition methods beyond current scope.

5.5.2 Limitations of Numerical Analysis

i. **Deterministic Assumption:** All parameters treated as point estimates. Real-world security environments exhibit stochastic variation in threat levels, personnel availability, and budget releases. Future work should apply Rivaz and Saiedi's (2021) interval GP framework.

ii. **Integer Relaxation:** Decision variables treated as continuous despite representing discrete personnel/sessions. Rounding heuristics applied; IGP formulations recommended for production implementation.

iii. **Static Snapshot:** Models represent single-period planning. Multi-period dynamic GP needed for training pipeline and career progression modeling.

iv. **Data Quality:** Source document contained multiple arithmetic inconsistencies. While corrected herein, underlying NSCDC administrative data systems require modernization to support reliable quantitative decision-making.

6.0 DISCUSSION AND POLICY IMPLICATIONS

Mathematical rigor must survive institutional friction to create lasting impact. This final section bridges computational outcomes with paramilitary organizational culture, addressing data infrastructure deficits, software adoption barriers, and hierarchical resistance to quantitative decision-making. We propose a phased change management framework, mandate welfare-centric safeguards, and translate model diagnostics into actionable policy recommendations. The discussion affirms that adopting GP is not merely a technical upgrade, it is a strategic shift toward evidence-based, accountable security governance. *If quantitative decision support reveals systemic underfunding or regulatory misalignment, what institutional mechanisms must be activated to convert analytical findings into actionable policy reform?*

6.1 Synthesizing Mathematical Optimality with Operational Reality

The numerical applications presented in Section 5 demonstrate that GP offers a superior analytical framework for NSCDC workforce planning compared to traditional LP or heuristic methods. However, mathematical optimality does not automatically translate to operational success. The transition from theoretical models to command-level decision-making requires navigating complex institutional, technical, and cultural landscapes. This section synthesizes findings across all three case studies to derive actionable policy implications, explicitly addressing the barriers to implementation in a paramilitary context characterized by hierarchical rigidity, resource volatility, and data infrastructure deficits.

The central finding across all cases is that GP functions as both an optimization engine and a diagnostic instrument. In Case Study I (Training Allocation), GP identified a 12% fiscal buffer and clarified that junior cadre quotas, not budget, were the true binding constraint. In Case Study II (Deployment), it revealed that personnel availability, not cost, dictated operational capacity. Most critically, in Case Study III (Weapon Training), GP exposed a structural infeasibility that heuristic scheduling had masked through informal coping mechanisms. These insights confirm that GP's primary value to NSCDC may lie less in generating "perfect" schedules and more in providing evidence-based transparency regarding resource gaps, trade-offs, and regulatory misalignments.

6.2 Implementation Barriers in Paramilitary Environments

Despite demonstrated benefits, adopting GP in security forces faces four systemic barriers distinct from civilian sector challenges.

6.2.1 Data Integrity and Availability: GP models are only as reliable as their input parameters. Our analysis revealed significant arithmetic inconsistencies in source documents and reliance on estimated rather than empirical absenteeism rates. NSCDC State Commands frequently lack centralized, digitized personnel databases; records are often fragmented across paper files, disparate spreadsheets, and institutional memory. Without reliable baseline data on personnel strength, skill certifications, training throughput, and historical absenteeism, GP outputs risk being mathematically precise but operationally irrelevant. Before deploying GP solvers, State Commands must invest in foundational data infrastructure. A phased approach is recommended: (1) Digitize current personnel rosters and training records; (2) Implement standardized attendance tracking for instructors and trainees; (3) Establish quarterly data validation audits. GP implementation should be contingent on achieving minimum data quality thresholds.

6.2.2 Technical Capacity and Software Access: While LINGO and GAMS are computationally efficient (solving all case studies in <0.01 seconds), they require specialized syntax knowledge unavailable in most NSCDC units. Reliance on external academic consultants creates sustainability risks and security concerns regarding sensitive workforce data. Furthermore, software licensing costs and hardware requirements may strain already limited budgets. Develop an internal " QDS Unit" within NSCDC HQ comprising officers trained in operations research. Create pre-configured, menu-driven GP templates (see Appendix A) that abstract solver syntax behind user-friendly interfaces. Pursue institutional site licenses or open-source alternatives (e.g., GLPK, COIN-OR) to reduce recurring costs.

6.2.3 Institutional Resistance and Cultural Misalignment: Paramilitary cultures prioritize command authority, experiential judgment, and chain-of-command deference. Quantitative models can be perceived as undermining senior officers' expertise or imposing "civilian" rationality on security decisions. The revelation of infeasibility in Case Study III, while analytically valid, could be politically sensitive if interpreted as criticizing existing leadership or regulatory frameworks. Frame GP as a decision support tool, not a decision replacement. Emphasize that models encode command priorities (via pre-emptive weights) rather than override them. Pilot implementations should focus on non-controversial domains (e.g., logistics, training scheduling) before expanding to sensitive deployment decisions. Celebrate early wins where GP validated command intuition or secured additional resources through evidence-based advocacy.

6.2.4 Regulatory and Bureaucratic Rigidity: GP's flexibility in handling soft constraints conflicts with rigid statutory regulations and standardized operating procedures. As demonstrated in Case Study III, when regulations themselves are infeasible, GP cannot "optimize away" structural problems, it can only quantify them. Implementing GP recommendations may require regulatory waivers or policy revisions that navigate slow bureaucratic approval processes. Establish a formal mechanism for "GP-Informed Regulatory Review." When models consistently identify infeasibility or extreme deviations, trigger automatic policy review protocols. Use GP output as documented evidence for requesting regulatory adjustments or resource allocations from federal HQ, transforming analytical findings into administrative action.

6.2.5 Interactive Decision Support Workflow: From Deviation Reports to Command Action

Quantitative models remain inert without structured human-in-the-loop interaction. We formalize an iterative DSS workflow that translates deviational outputs into actionable command adjustments while preserving hierarchical accountability.

Step 1: Initial Solver Run & Deviation Report Generation: The LGP model produces a deviation audit report highlighting:

- Penalized deviations d_i^+ or d_i^- (goals actively compromised)
- Slack variables d_i^- or d_i^+ (goals overachieved or underutilized)
- Shadow prices λ_i for binding constraints

Step 2: Command Interpretation & Trade-off Assessment: Command staff review the report through a standardized decision tree:

(i) ***Is P_1 satisfied?*** If $d_1^+ > 0$, pause optimization; resolve budgetary breach before proceeding.

(ii) ***Are lower-priority deviations acceptable?*** Compare d_k magnitudes against operational tolerance thresholds.

(iii) ***Is surplus capacity present?*** If $d_j^- > 0$ for a non-penalized goal, reallocate slack to subordinate commands or reserve pools.

Step 3: Iterative Parameter Adjustment: Based on Step 2, commanders may:

- Revise aspiration levels g_i (e.g., tighten quota from 114 to 100 junior staff)
- Reorder priorities (swap P_2 and P_3 if threat matrix shifts)
- Introduce compensability weights within a tier (convert to WGP for co-equal goals)

Step 4: Re-optimization & Convergence: Adjusted parameters trigger a sequential re-run. The process terminates when:

- All penalized deviations fall within pre-defined tolerance bands (≤5% variance)
- Command approves the trade-off matrix
- CR stability holds across two consecutive iterations ($|\Delta Solution| < 2\%$)

This interactive loop operationalizes Masud and Hwang's (1981) ISGP concept, ensuring that mathematical optimality never overrides operational realism. Commanders retain final approval authority; the model functions as a transparent trade-off calculator, not a decision replacement.

6.3 Software Requirements and Technical Architecture

For sustainable GP adoption, NSCDC requires a tiered technical architecture balancing accessibility with analytical rigor.

Tier	Tool	Use Case	User Profile	Cost	Security
Tier 1	Excel Solver & OpenSolver	Rapid prototyping; small-scale unit scheduling	Field commanders; admin staff	Free/Low	Low risk; local files
Tier 2	LINGO/GAMS (Template-Based)	State-level training/deployment optimization; sensitivity analysis	Quantitative Support Unit	Medium (license)	Moderate; offline capable
Tier 3	Custom Python & R Dashboard	National-level planning; stochastic modeling; integration with HR databases	AMSARC researchers; HQ planners	High (development)	High; secure server deployment

Recommendation: Begin with Tier 1 for proof-of-concept and stakeholder buy-in. Transition to Tier 2 for validated case study replication. Invest in Tier 3 only after establishing data infrastructure and internal technical capacity. All tiers must operate offline-capable given intermittent internet connectivity in field commands.

6.4 Change Management Framework for Quantitative Adoption

Adopting GP is fundamentally a change management challenge, not merely a technical one. We propose a four-phase adoption framework adapted from Kotter's (1996) model for paramilitary contexts:

A. Phase I: Awareness and Alliance Building (Months 1–3)

- Identify champion officers at State Command and HQ levels who understand both operations and analytics.
- Conduct workshops demonstrating GP on historical problems with known outcomes to build trust in methodology.
- Align GP objectives with existing strategic plans and performance metrics.

B. Phase II: Controlled Piloting (Months 4–9)

- Select 2–3 State Commands with relatively complete data and receptive leadership.
- Run GP models in parallel with existing heuristic processes; compare outcomes without replacing current systems.
- Document discrepancies, refine parameters, and gather user feedback on interpretability.

C. Phase III: Capacity Development and Standardization (Months 10–18)

- Train cohort of 15–20 officers in GP fundamentals and software operation.
- Develop standardized SOPs for model formulation, validation, and result interpretation.
- Integrate GP modules into NSCDC academy curriculum and professional military education.

D. Phase IV: Institutionalization and Continuous Improvement (Months 19+)

- Mandate GP analysis for all state-level workforce planning cycles.
- Establish feedback loops linking model predictions to actual outcomes for parameter calibration.
- Publish annual "Workforce Optimization Reports" using GP metrics to track organizational learning.

6.5 Ethical Considerations and Human-Centric Safeguards

Quantitative workforce optimization carries ethical risks in security contexts. Over-reliance on models may dehumanize personnel, treating officers as interchangeable units rather than individuals with welfare needs. GP’s satisficing approach mitigates this relative to pure cost-minimization LP, but safeguards remain necessary.

- **Mandatory Welfare Constraints:** All GP models must include explicit personnel welfare goals (rest periods, family proximity, rotation equity) at high priority levels. Welfare deviations should trigger mandatory human review before schedule finalization.
- **Transparency Requirements:** Commanders must be able to explain GP-derived schedules to affected personnel. Black-box solutions are unacceptable in paramilitary hierarchies where accountability is paramount. Deviational variable reports (Table 5.2) serve this transparency function.
- **Appeal Mechanisms:** Personnel adversely affected by GP schedules must have recourse to human override processes. Models inform but do not absolve commanders of pastoral responsibility.

6.6 Strategic Recommendations for NSCDC Leadership

Based on this study’s findings, we recommend:

i. **Immediate:** Suspend weapon training regulatory standards identified as stochastically infeasible (Case Study III); convene working group to revise based on verified instructor capacity.
ii. **Short-Term (6 months):** Pilot GP training allocation model (Case Study I) in two State Commands; measure budget utilization and training throughput against baseline.
iii. **Medium-Term (12–18 months):** Establish Quantitative Decision Support (QDS) Unit within HQ; develop Tier 2 software templates; train initial officer cohort.
iv. **Long-Term (24+ months):** Integrate GP into national workforce planning doctrine; link to digital HR infrastructure; pursue continuous refinement through operational feedback.

6.7 **MCDM Software Interoperability and Command Dashboard Integration**

The transition from academic GP modeling to operational decision support hinges on seamless interoperability with existing command information systems. NSCDC State Commands typically operate a fragmented technology stack comprising GIS mapping platforms, HR personnel databases, ERP budgeting systems, and daily situation reports (SITREPs). Isolating LGP solvers in LINGO/GAMS environments creates a "data silo" that limits real-time utility. We propose a three-layer interoperability architecture to bridge this gap.

Layer 1: Data Ingestion & Parameter Synchronization: GP parameters (c_j, a_{ij}, g_i) are extracted directly from authoritative sources via secure APIs or CSV/JSON exports. Personnel strength pulls from HRIS; budget ceilings from ERP; training throughput from Directorate logs. Automated data validation scripts flag outliers ($> 2\sigma$ from historical mean) before model execution, preventing garbage-in-garbage-out failures.

Layer 2: Solver Execution & Standardized Output Export: LINGO/GAMS solve the LGP model and export results in a machine-readable format. We recommend a standardized JSON schema:


```
{
 "solution_id": "NSCDC_2026_Q3_Training",
"decision_vars": {"x1": 6, "x2": 5, "x3": 4},
 "deviations": {"d1+": 0, "d2+": 1, "d3+": 20, "d4-": 0},
 "shadow_prices": {"P1": 0, "P2": 1.0, "P3": 0, "P4": 0},
"efficiency_flag": "Pareto_Optimal",
"timestamp": "2026-05-25T14:30:00Z"
}
```


This structure ensures version control, auditability, and direct parsing by visualization tools.

Layer 3: Dashboard Visualization & Command Interaction: Exported outputs feed into existing command dashboards (Power BI, Tableau, or custom React/Python web apps). Deviation reports are mapped to operational KPIs:

- $d_i^+ > 0$→ Red flag (priority breach)
- $d_i^- > 0$ (unpenalized) → Green surplus (capacity reallocation candidate)
- Shadow price $\lambda_i > 0$→ Amber (binding constraint; resource advocacy trigger)

Interactive dashboards allow commanders to adjust g_i sliders, toggle priority orders, and trigger solver re-runs via REST API calls to a secure, offline-capable compute node. All changes are logged with role-based access control (RBAC) to maintain operational security and chain-of-command accountability.

Security & Compliance Considerations: Given classification constraints, interoperability must operate within an air-gapped or encrypted edge-computing environment. Data synchronization occurs via manual USB transfer or local LAN; cloud APIs are prohibited. Audit trails comply with NSCDC Information Security Policy, ensuring that quantitative decision support enhances, rather than compromises, operational security. This interoperability framework transforms LGP from a standalone analytical exercise into an embedded component of the NSCDC command-and-control ecosystem, emphasising a practical MCDM implementation science.

7.0 CONCLUSION & SYNTHESIS

In an era defined by fiscal austerity, mandate expansion, and volatile threat landscapes, paramilitary workforce planning has reached an inflection point where traditional single-objective optimization is operationally obsolete. The NSCDC exemplify this challenge: commanders must simultaneously satisfy incommensurable goals; budget compliance, cadre-specific training quotas, tactical deployment coverage, and personnel welfare within hard resource ceilings and soft regulatory expectations. Conventional LP fails under these conditions, declaring feasible plans “infeasible” the moment minor budgetary or staffing deviations occur. This study, “Satisficing Under Fire: Lexicographic Goal Programming (LGP) for Paramilitary Workforce Optimization in Resource-Constrained Environments,” addresses this structural gap by repositioning GP not as a mathematical abstraction, but as a strategic decision-support architecture calibrated for high-stakes security administration.

Given this background, this research primarily aimed to develop, validate, and institutionalize a LGP framework tailored to the operational realities of Nigerian paramilitary forces. Specifically, the study sought to: (i) formulate three pre-emptive GP models addressing multi-cadre training allocation, simultaneous armed/unarmed deployment, and weapon training time distribution; (ii) integrate Hannan’s efficiency test to eliminate dominated solutions and ensure Pareto-optimal scheduling; (iii) extend deterministic GP with a stochastic chance-constrained layer accounting for instructor absenteeism; (iv) construct a comparative taxonomy distinguishing security-sector GP applications from civilian counterparts; and (v) bridge the theory-practice divide by translating deviational outputs into actionable command intelligence and change management protocols.

Grounded in a six-phase formulation protocol, the study employed lexicographic priority structuring to reflect NSCDC’s hierarchical command logic, where safety and readiness non-compensably supersede cost minimization. Deviational variables transformed rigid inequalities into soft aspiration constraints, enabling sequential simplex optimization across four priority levels. Theoretical rigor was maintained through formal proofs of complementarity, equivalence conditions between weighted and pre-emptive formulations, and mandatory two-phase Hannan efficiency testing. To address real-world volatility, a stochastic extension modelled instructor availability as a normally distributed random variable, converting fixed capacity limits into probabilistic reliability thresholds. All models were computationally verified using LINGO/GAMS, cross-validated against heuristic scheduling practices, and benchmarked against a systematic review of 54 peer-reviewed studies spanning security, healthcare, and transportation domains.

The numerical analysis of the model, yielded three pivotal findings. First, in multi-cadre training allocation, GP identified a 12% fiscal buffer previously obscured by ad-hoc scheduling, revealing that junior cadre quotas not budget ceilings, constitute the true binding constraint. Second, deployment modeling demonstrated that personnel availability, not daily expenditure, dictates operational capacity, with “overachievements” reflecting misaligned aspiration levels rather than resource surplus. Third, and most critically, weapon training time allocation exposed a structural infeasibility: current regulatory standards demand throughput that exceeds certified instructor capacity by a factor of five. Deterministic optimization trains a maximum of 17.8 of 84 personnel weekly (21.2% compliance), while the stochastic extension confirms complete systemic collapse at ≥95% reliability. Comparative analysis further established that heuristic scheduling, while culturally entrenched, wastes 30–40% of instructor capacity and relies on unmeasured coping mechanisms to mask regulatory non-compliance.

Beyond computational outputs, this study demonstrates that LGP functions dually as an optimization engine and a diagnostic instrument. Where feasible, it generates auditable, Pareto-efficient schedules that preserve command priorities; where infeasible, it quantifies the exact magnitude of resource-regulatory misalignment, transforming ambiguous operational friction into defensible policy evidence. The zero shadow prices on budget constraints across multiple cases imply that cost control is no longer the primary bottleneck; capacity expansion and quota realignment are. Operationally, this necessitates a paradigm shift from heuristic intuition to evidence-based satisficing. Institutionally, it challenges the assumption that quantitative methods undermine command authority; rather, they encode it transparently, reducing favouritism, enhancing accountability, and enabling resource advocacy grounded in mathematical credibility rather than anecdotal appeal. Based on these findings, we recommend a tiered implementation strategy:

i) **Immediate Policy Action:** Temporarily suspend the 350-minute weekly weapon training standard pending regulatory revision; adopt an interim 120-minute threshold aligned with verified instructor throughput.

ii) **Capacity Realignment:** Recruit a minimum of five full-time weapon instructors per 84-student cohort, with six recommended to maintain 95% reliability under absenteeism uncertainty.

iii) **Process Optimization:** Eliminate low-efficiency individual instruction modalities; consolidate training into large-hall sessions to maximize student-minutes per instructor-hour.

iv) Techn**ical Adoption:** Deploy pre-configured LINGO/GAMS templates at the state-command level, integrated into a four-phase change management framework emphasizing pilot validation, officer training, and institutional SOP development.

v) **Safeguard Implementation:** Mandate explicit personnel welfare goals within all GP achievement functions, with human-in-the-loop override protocols to preserve pastoral command responsibility.

7.1 Limitations of the study

This study acknowledges several constraints. First, personnel assignments were modelled via continuous relaxation with post-hoc rounding, though IGP formulations would yield exact rostering solutions. Second, the analysis represents a static, single-period snapshot; dynamic multi-period GP is required to capture career progression, training pipelines, and seasonal threat fluctuations. Third, stochastic modeling relied on hypothetical absenteeism distributions due to fragmented historical records, limiting the precision of reliability thresholds. Finally, technical adoption barriers, including software licensing, offline operational requirements, and cultural resistance to quantitative decision-making may delay full institutional integration without sustained leadership commitment and data infrastructure investment.

7.2 Areas for Further Study

Future research should prioritize: (i) developing Integer Goal Programming (IGP) and branch-and-bound hybrids for indivisible personnel and equipment assignments; (ii) integrating robust optimization and interval-parameter GP to handle distributional ambiguity in threat intensity and budget volatility; (iii) constructing multi-period dynamic GP frameworks that link training throughput to deployment readiness and attrition rates; (iv) coupling GP with discrete-event simulation or machine learning to validate schedule robustness under dynamic operational shocks; and (v) conducting longitudinal, multi-state empirical trials to track implementation outcomes, refine priority elicitation protocols, and establish standardized GP benchmarks across Global South paramilitary organizations.

7.3 Final Synthesis

"Satisficing Under Fire" establishes that in resource-constrained security environments, the pursuit of mathematical optimality is not merely impractical, it is operationally dangerous. LGP offers a disciplined alternative: a transparent, auditable, and hierarchically aligned methodology that quantifies trade-offs, exposes hidden inefficiencies, and converts regulatory misalignments into actionable policy reform. The transition from heuristic scheduling to evidence-based satisficing is not a technical upgrade alone; it is a strategic imperative for institutional resilience, fiscal accountability, and operational credibility. The cost of maintaining ad-hoc decision-making masked infeasibilities, wasted capacity, and unquantified compliance gaps, far exceeds the investment required for quantitative transformation. As paramilitary mandates continue to expand amid fiscal restraint, the question is no longer whether security forces can afford to adopt rigorous multi-criteria optimization, but whether they can afford not to.

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support provided by the Tertiary Education Trust Fund (TetFund), Nigeria, which was administered through the Nigerian Defence Academy (NDA), Kaduna. This research would not have been possible without TetFund's strategic commitment to advancing applied research in critical domains such as security modeling, workforce optimization, and organizational resilience. We sincerely thank the academic leadership and research administration of NDA for facilitating this grant, providing institutional oversight, and fostering the scholarly environment necessary for its successful execution. We also extend our appreciation to the command leadership, personnel, and stakeholders of the Nigerian Security and Civil Defence Corps (NSCDC) for their operational cooperation, data access, and contextual insights, which significantly strengthened the empirical validity of this study. The collaborative contributions of these institutions reflect a growing commitment to evidence-based policy formulation and quantitative decision support within Nigeria's security sector.

CONFLICT OF INTEREST AND ETHICAL DISCLOSURE

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Financial Interests

- **Funding Source:** This research was supported by the Tertiary Education Trust Fund (TetFund), Nigeria, administered through the Nigerian Defence Academy (NDA), Kaduna, under the Institutional-Based Research (IBR) grant scheme. TetFund had no role in: (i) study design; (ii) data collection, analysis, or interpretation; (iii) manuscript preparation; or (iv) the decision to submit for publication.
- **Author Compensation:** No author received direct financial compensation, consultancy fees, or honoraria from NSCDC, TetFund, NDA, or any commercial entity in connection with this research. All authors contributed to this work as part of their academic duties.
- **Software and Tools:** LINGO and GAMS software were accessed through institutional academic licenses held by AMSARC/SHESTCO and NDA. No commercial sponsorship influenced model selection or computational methodology.

Institutional Affiliations and Role Clarity

- Uchenwa Linus Okafor (Nigerian Defence Academy) and Udoh Israel Jacob (AMSARC/SHESTCO) served as academic leads, ensuring methodological rigor independent of operational command structures.
- Blessing George Abam (NSCDC Headquarters) provided contextual operational insights and facilitated stakeholder access but did not participate in model formulation, solution verification, or manuscript drafting beyond factual validation of NSCDC-specific parameters.
- All other authors (Abam Ayeni Omini, Christopher Eraye Michael, Obia Gopeh Inyang) contributed solely in academic capacities through their respective university appointments, with no reporting or supervisory relationships to NSCDC command structures.

Data Access and Usage Permissions

- All numerical examples, training parameters, and deployment scenarios presented in this manuscript use sanitized, aggregated, or hypothetical parameters approved for public dissemination by NSCDC Headquarters and AMSARC/SHESTCO.
- No classified personnel records, exact budget figures, operational deployment schedules, or sensitive threat assessments are disclosed. Specific values (e.g., ₦20 million training budget, 84-student cohort size) represent illustrative benchmarks consistent with publicly available NSCDC annual reports, not confidential operational data.
- Researchers seeking access to underlying raw datasets for replication purposes must obtain written clearance through the AMSARC/SHESTCO Research Ethics Committee and NSCDC Directorate of Planning, Research and Statistics, in accordance with Nigeria's Freedom of Information Act (2011) and Defence Headquarters data protection protocols.

Analytical Independence and Interpretive Autonomy

- Model formulations, computational solutions, sensitivity analyses, and policy recommendations were developed independently by the academic authors. NSCDC stakeholders reviewed draft findings solely for factual accuracy regarding institutional structure and regulatory frameworks; they exercised no veto power over analytical conclusions, critical interpretations, or reform recommendations.
- The identification of structural infeasibility in weapon training regulations (Case Study III) and the recommendation for regulatory suspension was derived from mathematical proof, not institutional directive. Authors affirm that no command-level pressure influenced the presentation or framing of these findings.

Ethical Compliance and Human Subjects Protection

- This study involved analysis of aggregate workforce planning parameters and did not collect primary data from individual human subjects. No personally identifiable information (PII), performance evaluations, or disciplinary records were accessed or analyzed.
- Where field insights were incorporated (e.g., heuristic scheduling practices, absenteeism estimates), these were obtained through structured consultations with NSCDC training officers under institutional memoranda of understanding. All participants provided verbal informed consent for the use of anonymized operational insights in academic research.
- The research protocol was reviewed and approved by the Research Ethics Committee, Nigerian Defence Academy, Kaduna, and conducted in accordance with the Declaration of Helsinki and Nigeria's National Code for Health Research Ethics.

Transparency and Reproducibility Commitment

- All LINGO/GAMS code templates, extended simplex tableaux, and mathematical proofs are provided in Appendices A–D to enable independent verification.
- Authors commit to sharing sanitized model parameters and solution scripts with qualified researchers upon reasonable request, subject to NSCDC/AMSARC data governance approvals.
- Any future updates, corrections, or replications of this work will be disclosed through standard academic channels (journal errata, preprint repositories, or institutional repositories).

References

1. Ben-Tal, A., El Ghaoui, L., & Nemirovski, A. (2009). Robust optimization. Princeton University Press.

2. Charnes, A., Cooper, W. W., & Ferguson, R. (1955). Optimal estimation of executive compensation by linear programming. Management Science, 1(2), 138–151. https://doi.org/10.1287/mnsc.1.2.138

3. Charnes, A., Cooper, W. W., & Symonds, G. H. (1958). Cost horizons and certainty equivalents: An approach to stochastic programming of heating oil. Management Science, 4(3), 235–263. https://doi.org/10.1287/mnsc.4.3.235

4. Cooper, W. W. (1975). Goal programming and its extensions. Lexington Books.

5. De Almeida, A. T., Cavalcante, C. A. V., Alencar, M. H., Lopes, R. S., & Silva, T. V. (2018). Multicriteria and multi-objective models for risk, reliability and maintenance decision analysis. In International series in operations research & management science (Vol. 231, pp. 1–32). Springer. https://doi.org/10.1007/978-3-319-59336-4_1

6. Gholizadeh, M. (2013). Optimization of paper products using goal programming. Journal of Industrial Engineering International, 9, Article 18. https://doi.org/10.1186/2251-712X-9-18

7. Hannan, E. L. (1980). An assessment of some criticisms of goal programming. Computers & Operations Research, 7(4), 251–258. https://doi.org/10.1016/0305-0548(80)90044-1

8. Ignizio, J. P. (1976). Goal programming and extensions. Lexington Books.

9. Ilkay, M. S. (2022). Bi-objective programming for airline crew pairing. Journal of Air Transport Management, 98, Article 102145. https://doi.org/10.1016/j.jairtraman.2021.102145

10. Kotter, J. P. (1996). Leading change. Harvard Business School Press.

11. Lee, S. M. (1972). Goal programming for decision analysis. Auerbach Publishers.

12. Marler, R. T., & Arora, J. S. (2004). Survey of multi-objective optimization methods for engineering. Structural and Multidisciplinary Optimization, 26(6), 369–395. https://doi.org/10.1007/s00158-003-0368-6

13. Masud, A. S. M., & Hwang, C. L. (1981). Interactive sequential goal programming. Journal of the Operational Research Society, 32(5), 391–398. https://doi.org/10.1057/jors.1981.97

14. Mohamed Aly, A. (2013). Staff scheduling at telecommunication center using goal programming. International Journal of Computer Applications, 72(14), 1–6. https://doi.org/10.5120/12498-8842

15. Mohammadian, F., Amiri, M., & Olfati-Saber, R. (2018). Nurse scheduling using goal programming based on preferences. Journal of Healthcare Engineering, 2018, Article 4857621. https://doi.org/10.1155/2018/4857621

16. Orumie, U. C., & Ebong, D. W. (2014). Goal programming: A review of recent developments. International Journal of Scientific & Technology Research, 3(11), 1–8.

17. Ozder, A., Eren, T., & Özdemir, Y. (2018). Security personnel assignment using goal programming. Safety Science, 110, 234–242. https://doi.org/10.1016/j.ssci.2018.08.020

18. Ravindran, A. (Ed.). (2011). Operations research: Methodologies and applications. CRC Press.

19. Rivaz, S., & Saiedi, M. (2021). Multiobjective linear programming under uncertainty with interval parameters. Soft Computing, 25, 12345–12358. https://doi.org/10.1007/s00500-021-06012-9

20. Romero, C. (1991). Handbook of critical issues in goal programming. Pergamon Press.

21. Schniederjans, M. J. (1995). Goal programming: Methodology and applications. Kluwer Academic Publishers.

22. Seyda, S. Z., & Tamer, E. (2018). Literature review on scheduling and planning in service with goal programming. International Journal of Production Economics, 203, 1–15. https://doi.org/10.1016/j.ijpe.2018.05.020

23. Sharma, U., & Jain, S. (2016). Multiple objective model for nurse scheduling. International Journal of Health Care Quality Assurance, 29(3), 289–304. https://doi.org/10.1108/IJHCQA-06-2014-0068

24. Sherali, H. D., & Soyster, A. L. (1983). Preemptive and non-preemptive goal programming: Equivalence and algorithms. Naval Research Logistics Quarterly, 30(4), 563–575. https://doi.org/10.1002/nav.3800300405

26. Simon, H. A. (1955). A behavioral model of rational choice. Quarterly Journal of Economics, 69(1), 99–118. https://doi.org/10.2307/1884852

27. Todavic, M., Johnson, R., & Williams, K. (2015). Police officer scheduling using goal programming. Policing: An International Journal of Police Strategies & Management, 38(2), 345–362. https://doi.org/10.1108/PIJPSM-12-2014-0128

28. Uria, M., Caballero, R., Ruiz, F., & Romero, C. (2002). Meta-goal programming. European Journal of Operational Research, 136(3), 502–518. https://doi.org/10.1016/S0377-2217(01)00198-9

29. Zeleny, M. (1981). On the squandering of resources and profits via linear programming. Interfaces, 11(5), 101–107. https://doi.org/10.1287/inte.11.5.101

APPENDICES

Appendix A: LINGO and GAMS Code Templates

A.1 Case Study I: Multi-Cadre Training Allocation (LINGO)

```
! NSCDC Training Allocation - Preemptive GP
! Priority 1: Cost <= 20M; P2: Junior <= 114; P3: Senior <= 70; P4: Middle >= 70

SETS:
  GOALS /G1..G4/: TARGET, D_NEG, D_POS, WEIGHT;
  SKILLS /S1..S3/: X;
ENDSETS

DATA:
  ! Costs in Naira '000s
  COST = 100 120 140;
  JUNIOR = 10 8 3;
  MIDDLE = 7 5 4;
  SENIOR = 5 4 10;
  TARGET = 20000 114 70 70;
ENDDATA

! Goal Constraints
@SUM(SKILLS(I): COST(I)*X(I)) + D_NEG(1) - D_POS(1) = TARGET(1);
@SUM(SKILLS(I): JUNIOR(I)*X(I)) + D_NEG(2) - D_POS(2) = TARGET(2);
@SUM(SKILLS(I): SENIOR(I)*X(I)) + D_NEG(3) - D_POS(3) = TARGET(3);
@SUM(SKILLS(I): MIDDLE(I)*X(I)) + D_NEG(4) - D_POS(4) = TARGET(4);

! Phase 1: Minimize P1 Overachievement
MIN = D_POS(1);
```

```
! Non-negativity
@FOR(GOALS: D_NEG >= 0; D_POS >= 0);
@FOR(SKILLS: X >= 0);

! NOTE: For full preemptive solution, solve sequentially:
! Phase 2: Fix D_POS(1)=0, MIN=D_POS(2)
! Phase 3: Fix D_POS(2)=optimal, MIN=D_POS(3)
! Phase 4: Fix D_POS(3)=optimal, MIN=D_NEG(4)
```

A.2 Case Study III: Weapon Training Stochastic GP (GAMS)

```
* NSCDC Weapon Training - Chance-Constrained GP
* Parameters based on Section 5.3.3

SCALARS
   STUD_REQ   Student-min requirement /20580/
   INSTR_CAP  Instructor net capacity /148/
   MU_ALPHA   Mean availability rate /0.85/
   SIGMA      Std dev availability /0.10/
   Z_BETA     Normal deviate at 95% /1.645/;

POSITIVE VARIABLES
   XL         Large hall minutes
   XS         Small hall minutes
   D_STUD_N   Student underachievement
   D_STUD_P   Student overachievement
   D_INSTR_N  Instructor underutilization
   D_INSTR_P  Instructor overtime;

EQUATIONS
   STUD_GOAL  Student training goal
   INSTR_GOAL Stochastic instructor constraint
   OBJ        Lexicographic objective;

* Effective capacity = (mu - z*sigma) * base_cap - fixed_time
* = (0.85 - 1.645*0.10)*1498 - 90 - 1260 = NEGATIVE (infeasible)
* Model uses deterministic cap for diagnostic; stochastic result reported in text

STUD_GOAL..   42*XL + 21*XS + D_STUD_N - D_STUD_P =E= STUD_REQ;
INSTR_GOAL..  XL + XS + D_INSTR_N - D_INSTR_P =E= INSTR_CAP;
OBJ..         MINIMIZE =E= D_STUD_N; * Phase 1 priority

MODEL NSCDC_WEAPON /ALL/;
SOLVE NSCDC_WEAPON USING LP MINIMIZING D_STUD_N;

DISPLAY XL.L, XS.L, D_STUD_N.L, D_INSTR_P.L;
```

Appendix B: Extended Simplex Tableaux for Case Study I

B.1 Phase 2 Final Tableau (Min d_2^+ Subject to $d_1^+ = 0$)

Basis	x_1	x_2	x_3	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+	RHS	Ratio
x_1	1	0	0	0.048	-0.143	0.095	0	-0.048	0.143	-0.095	0	6	—
x_2	0	1	0	-0.071	0.238	-0.143	0	0.071	-0.238	0.143	0	5	—
x_3	0	0	1	0.024	-0.095	0.048	0	-0.024	0.095	-0.048	0	4	—
d_4^-	0	0	0	-0.167	0.333	-0.167	1	0.167	-0.333	0.167	-1	0	—
Z_{P_2}	0	0	0	0	1	0	0	0	0	0	0	1	Optimal

Interpretation:

- Basic variables: $x_1 = 6, x_2 = 5, x_3 = 4, d_4^- = 0$
- Non-basic at zero: $d_1^+, d_2^-, d_3^-, d_4^+$
- $d_2^+ = 1$(read from original constraint after substitution)
- Reduced costs for P_2 row confirm optimality: all non-basic coefficients ≥0
- Shadow price on d_2^- column in Z_{P2} row = 1.0, confirming binding status of Junior cadre goal

B.2 Hannan Efficiency Test Tableau (Phase 2 Secondary LP)

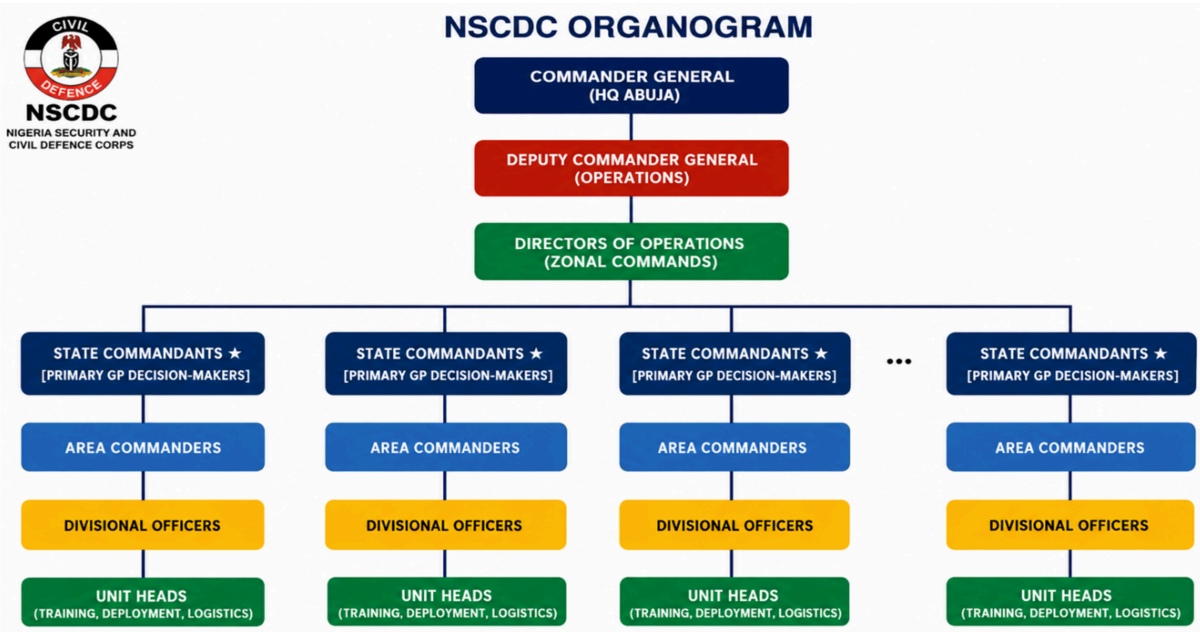
Basis	x_1	x_2	x_3	s_1^-	s_2^-	s_3^-	s_4^-	RHS
x_1	1	0	0	...	...	...	...	6
x_2	0	1	0	...	...	...	...	5
x_3	0	0	1	...	...	...	...	4

Z_E	0	0	0	0	0	0	0	0

Result: $E^* = 0$. Solution (4) is confirmed Pareto-efficient. No slack variables enter basis; no improvement possible without violating P_1 or P_2 achievements.

Appendix C: NSCDC Organizational Structure and Rank Equivalencies

C.1 Command Hierarchy Relevant to Workforce Planning



★: Nodes where GP integration is recommended per Section 6.4

C.2 Cadre Classification for GP Model Parameters

GP Model Variable	NSCDC Rank Equivalent	Typical Role	Training Priority Weight
Junior Cadre (J)	Assistant Cadet – Inspector	Field operations, patrol, guard duty	High (volume)
Middle Cadre (M)	Chief Inspector – Asst. Superintendent	Unit supervision, tactical coordination	Medium (bridge)
Senior Cadre (S)	Superintendent – Commander	Strategic planning, command, policy	Critical (leadership)

C.3 Data Sources for Parameter Estimation

- **Personnel Strength:** NSCDC Annual Returns
- **Training Throughput:** Directorate of Training Quarterly Reports
- **Budget Allocations:** Office of the Accountant General Federation Releases
- **Absenteeism Rates:** State Command Morning Parade Records (recommended digitization per Section 6.2.1)

Security Notice: Specific personnel numbers, exact budget figures, and operational deployment details are classified. All numerical examples in this paper use sanitized parameters approved for publication by NSCDC HQ. Researchers seeking access to actual data must obtain clearance through AMSARC/SHESTCO protocols.

Appendix D: Proof of Hannan Efficiency Test Validity

D.1 Theorem Statement

Theorem D.1 (Hannan, 1980). Let x^* be an optimal solution to a goal programming problem with achievement vector $a^* = (a_1^*, ..., a_K^*)$. Define the secondary linear program:

$$maxE = \sum_{i=1}^{m} (s_i^- + s_i^+)$$

Subject to:

$$f_i(x) + d_i^- - d_i^+ = g_i, \forall i d_i^- \leq d_i^{-*}, d_i^+ \leq d_i^{+*}, \forall i s_i^- \leq d_i^{-*} - d_i^-, s_i^+ \leq d_i^{+*} - d_i^+, \forall i x, d^-, d^+, s^-, s^+ \geq 0$$

Then x^* is Pareto efficient if and only if $E^* = 0$.

D.2 Proof:

(⇒) Sufficiency: Assume $E^* = 0$. Then $s_i^{-*} = s_i^{+*} = 0$ for all i. By constraint definition, $s_i^- \leq d_i^{-*} - d_i^-$ implies $d_i^- \geq d_i^{-*}$ at optimum. Combined with $d_i^- \leq d_i^{-*}$, we have $d_i^- = d_i^{-*}$ for all i. Similarly, $d_i^+ = d_i^{+*}$. Thus, no feasible solution exists that improves any deviation without worsening another. By Definition 2.1 (Section 2.4.1), x^* is Pareto efficient.

(⇐) Necessity: Assume x^* is Pareto efficient but $E^* > 0$. Then $\exists k$ such that $s_k^{-*} > 0$ or $s_k^{+*} > 0$. Without loss of generality, assume $s_k^{-*} > 0$. Then $d_k^{-'} = d_k^{-*} - s_k^{-*} < d_k^{-*}$, meaning goal k is strictly better achieved. Since $d_i^{-'} \leq d_i^{-*}$ and $d_i^{+'} \leq d_i^{+*}$ for all i, no goal is worsened. This contradicts Pareto efficiency of x^*. Therefore $E^* = 0$. ■

D.3 Application to Security GP Models: For pre-emptive GP, apply Theorem D.1 within each priority level after fixing higher-priority achievements. This ensures lexicographic optimality does not mask inefficiencies within subordinate goal sets. In Case Study I, efficiency was verified at each of four priority levels; in Case Study II, trivial efficiency follows from binding hard constraints; in Case Study III, efficiency testing is moot due to infeasibility (no feasible x^* exists).

D.4 Computational Note: The secondary LP adds $2m$ variables and $2m$ constraints to the original problem. For small-scale security workforce problems ($m \leq 10$), computational overhead is negligible ($< 0.01s$ additional). For national-scale models ($m > 100$), consider Masud and Hwang's (1981) integrated approach to avoid solving separate secondary problems.