

Original Research Article

NUMERICAL SIMULATION OF HYDROMAGNETIZED BOUNDARY LAYER FLOW EQUATION WITH VISCOUS DISSIPATION EFFECT ON A PLATE UNDER VARIABLE TEMPERATURE

ABIALA I. O¹, AROLOYE J.S¹, LAWAL S.A^{1*}, ADEOYE A.S²

¹ Department of Mathematics, University of Lagos, Nigeria.

² Department of Mathematical Science, Achievers University, Nigeria.

Correspondence should be addressed to Lawal Samsudeen Adewole: sadewole93000@gmail.com

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Abstract: With variable temperature and viscous dissipation, this paper considers a classical problem of an MHD boundary layer flow over a rectangular plate in a uniform stream of fluid. By using similarity transformation, the governing nonlinear PDEs have been transformed into a set of ODEs. Due to the presence of viscous dissipation, conversion of mechanical energy to thermal energy results in changes in temperature and other properties in the fluid. The effects of Prandtl number, Eckert number and temperature index coefficient n are discussed and graphically presented. It is found that as the Eckert number increases, the temperature distribution also increases, while there exists an inverse variation with the Prandtl number.

Keywords: Boundary Layer Flow, Plate, Eckert Number, Similarity Transformation, Variable Temperature, Viscous Dissipation.

List of symbols

B_0	Strength of magnetic field ($\text{Kgs}^{-2} \text{A}^{-1}$)
C_p	Specific Heat Capacity ($\text{JKg}^{-1}\text{K}^{-1}$)
M	Magnetic field parameter
Ec	Eckert number
q_r	Heat flux
Pr	Prandtl number

T_o	Surface temperature (K)
T_∞	Free stream temperature (K)
U	Fluid velocity
u, v	Velocity component corresponding to horizontal and vertical direction (ms^{-1})
σ	Electrical Conductivity (sm^{-1})
α	Thermal diffusivity
ν	Kinematic Viscosity (m^2s^{-1}).
η	Similarity variable
$f(\eta)$	Non-dimensional displacement parameter
$\theta(\eta)$	Non-dimensional temperature parameter

INTRODUCTION

On a rectangular flat plate, laminar flow is being extensively researched in both analytical and experimental ways.

Brinkman [1] happened to be the first one to carry out a study on viscous dissipation. Tyagi [7] investigated the impact of viscous dissipation effects on laminar fluid flow with uniform wall temperatures and forced convection in cylindrical tubes. Risbeck, Chen, and Armaly [3] worked on Laminar mixed convection over horizontal flat plates with power-law variation in surface temperature. An analysis was performed to study the heat transfer characteristics of laminar mixed convective boundary-layer flow over a semi-infinite horizontal flat plate with non-uniform surface temperatures. The surface temperature was assumed to vary as a power of the axial coordinate measured from the leading edge of the plate. A nonsimilar mixed convection parameter χ and a pseudo-similarity variable η were introduced to cast the governing boundary layer equations and their boundary conditions into a system of dimensionless equations, which were solved numerically by a weighted finite-difference method. In 2005, Emad M. Abo-Eldahab and Mohamed A. El Aziz took a look at Viscous dissipation and Joule heating effects on MHD-free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ion-slip currents. In 2006, Sen and Arora worked on the stability of laminar boundary-layer flow over a flat plate with a compliant surface. Qussai Jihad Abdul-Ghafour in 2011 obtained a general velocity profile for a laminar flow over a flat plate with zero incidence by employing a new boundary condition to the other available boundary conditions. The general velocity profile was mathematically simple and nearest to the exact solution. Also, other related values, boundary layer thickness, displacement thickness, momentum thickness and coefficient of friction were nearest to the exact solution compared with other corresponding values for other researchers. Gireesha et al [13] examined the effect of viscous dissipation and heat source on the flow and heat transfer of dusty fluid over an unsteady stretching sheet. Their paper

investigated the problem of hydrodynamic boundary layer flow and heat transfer of a dusty fluid over an unsteady stretching surface. The study considered the effects of frictional heating (viscous dissipation) and internal heat generation or absorption. The basic equations governing the flow and heat transfer were reduced to a set of non-linear ordinary differential equations by applying suitable similarity transformations. The transformed equations were numerically solved by the Runge-Kutta-Fehlberg-45 order method. An analysis is carried out for two different cases of heating processes, namely, variable wall temperature (VWT) and variable heat flux (VHF). The effects of various physical parameters, such as the magnetic parameter, the fluid-particle interaction parameter, the unsteady parameter, the Prandtl number, the Eckert number, the number density of dust particles, and the heat source/sink parameter on velocity and temperature profiles are shown in several plots. The effects of the wall temperature gradient function and the wall temperature function were tabulated and discussed.

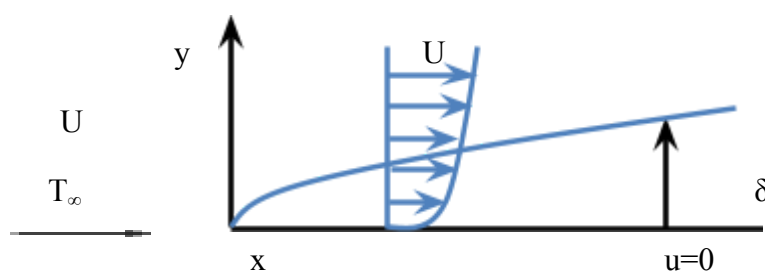
Satish Desale and Pradhan in 2015 looked into Numerical Solution of Boundary Layer Flow Equation with Viscous Dissipation Effect Along a Flat Plate with Variable Temperature. They presented a paper considering the classical problem of hydrodynamic and thermal boundary layers over a flat plate in a uniform stream of fluid with viscous dissipation effect and variable plate temperature. Using a similarity variable, the governing nonlinear partial differential equations were transformed into a set of coupled nonlinear ordinary differential equations and solved numerically by the finite difference method along with Newton's linearization approximation, namely the Keller box method. Due to viscous dissipation, conversion of mechanical energy to thermal energy results in temperature variation in the fluid and variation of fluid properties. A discussion was provided for the effects of Eckert number (Ec), Prandtl number (Pr) and temperature power coefficient n on two-dimensional flow. It was observed that as Ec increases, the temperature distribution increases, whereas as Pr increases, the temperature distribution decreases for variable temperature. In 2018, Syazwani Mohd Zokri et al. studied Influence of viscous dissipation on the flow and heat transfer of a Jeffrey fluid towards a horizontal circular cylinder with free convection: A numerical study. Their work focused on the numerical solution of free convection boundary layer flow past a horizontal circular cylinder in non-Newtonian Jeffrey fluid. The impact of viscous dissipation was discussed. [12] studied the impacts of temperature-dependent viscosity and variable Prandtl number on forced convective Falkner-Skan flow of Williamson nanofluid. In 2023, Numerical simulation of laminar flow and free convection Heat transfer from an isothermal vertical flat plate was studied by Belhocine, Stojanovic and Abdullah[4]. They presented a numerical study of the phenomenon of laminar natural convection in a vertical plate, whose wall was maintained at a constant temperature. It was assumed that the boundary layer problem was initially given in a two-dimensional flow, even though the physical properties of the fluid were considered to be constant except for the density change with the temperature. The governing equations of the model were transformed and simplified into a non-linear system of ordinary differential equations (ODE) through the use of similarity variables, which we were able to solve numerically using the Runge-Kutta method. This method had better

opted for the numerical resolution of this system, which was developed in FORTRAN code on the computer. The numerical results of the model were presented in tabular form and the velocity and temperature profiles for various Prandtl numbers were analyzed and depicted graphically. In 2024, Nidhal Ben Khedher et al studied the viscous dissipation effect on the amplitude and oscillating frequency of heat transfer and electromagnetic waves of magnetically driven fluid flow along the horizontal circular cylinder. The significant importance of the present research was to remove the extreme temperature along the magnetically driven horizontal circular cylinder. The induced electromagnetic field was applied around the surface of the cylinder. The main novelty of current research was to control the thermal and magnetic boundary layer in the presence of viscous dissipation and an induced electromagnetic field. The dimensional mathematical form was developed with defined boundary conditions. The dimensional equations were transformed into dimensionless equations to generate physical factors. It was found that fluid velocity improves significantly as the buoyancy force increases around each position. It was noticed that the increasing oscillations in heat transfer are sketched for the maximum choice of Prandtl number. It was found that the maximum oscillations in current density were obtained for each Eckert parameter. It was noticed that a significant distribution in the temperature profile was obtained in the presence of viscous dissipation and a magnetic field. In the same year, Abubakar Usman [9] examined viscous dissipation, magnetic field and thermal radiation on the systematic flow of Casson fluid with gyrotactic microorganisms. To examine the pertinent flow parameters, a numerical approach called the spectral relaxation method (SRM) was employed. This SRM approach employed the basic Gauss-Seidel method to decouple and discretize the set of differential equations. The choice of this approach was due to its consistency and accuracy. The viscous dissipation parameter (Ec) was found to enhance fluid temperature, velocity and boundary layers (thermal and momentum boundary layer). The strong opposition of the magnetic parameter gave rise to the Lorentz force which dragged the fluid flow within the boundary layer. The nanoparticles were found to have a great effect on the gyrotactic microorganisms.

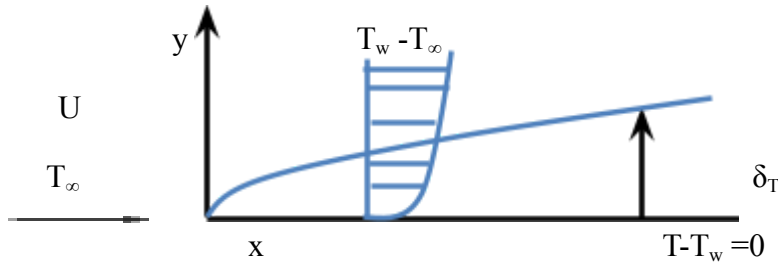
In this paper, [14] is improved by adjusting the magnetic parameter and viscous term to study the responses of dissipation, Prandtl number and Eckert number.

PROBLEM AND ITS GOVERNING EQUATIONS

Here, the magnetohydrodynamic boundary layer flow of an incompressible fluid through a flat plate with uniform T_w is considered. The effects of viscous dissipation, radiation and joule heating on the fluid flow are checked.



(a) velocity profile



(b) Temperature profile

Fig.1: Transmission of Heat in a Boundary Layer

The governing equations are as follows:

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum Equation:

$$\underbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}}_{\text{Convective acceleration in } x - \text{direction}} = \underbrace{\nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)}_{\text{Viscous term}} - \underbrace{\frac{\sigma}{\rho} B_o^2 \left(u + v \frac{\partial u}{\partial y} \right)}_{\text{Magnetic field}} \quad (2)$$

Energy equation:

$$\underbrace{u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}}_{\text{Convective Heat Transport term}} = \underbrace{\alpha \frac{\partial^2 T}{\partial y^2}}_{\text{Diffusion term}} + \underbrace{\frac{\nu}{C_p} \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 \right]}_{\text{Viscous Dissipation}} + \underbrace{\frac{1}{\sigma \rho C_p} B_o^2 u^2}_{\text{Joule heating}} - \underbrace{\frac{1}{\sigma \rho C_p} \frac{\partial}{\partial y} q_r}_{\text{Radiative term}} \quad (3)$$

Using Rosseland approximation, $q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$ for σ^* and k^* are Stefan-Boltzmann and absorption constants.

Taking $T^4 = 4T_\infty^3 T - 3T_\infty^4$ upon expansion of T^4 using Taylor's series around T_∞ .

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$$

$$\frac{\partial}{\partial y} q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial^2 T^4}{\partial y^2}$$

From (3),

$$\begin{aligned} -\frac{1}{\sigma\rho C_p}\frac{\partial}{\partial y}q_r &= -\frac{1}{\sigma\rho C_p}\left(-\frac{4\sigma^*}{3k^*}\frac{\partial^2 T^4}{\partial y^2}\right) \\ &= \frac{4}{3\sigma\rho C_p}\frac{\sigma^*}{k^*}\frac{\partial^2(4T_\infty^3T-3T_\infty^4)}{\partial y^2} \\ &= \frac{4}{3\sigma\rho C_p}\frac{\sigma^*}{k^*}\frac{\partial^2(4T_\infty^3T)}{\partial y^2} \\ &= \frac{4}{3\sigma\rho C_p}\frac{\sigma^*}{k^*}\frac{4T_\infty^3\partial^2 T}{\partial y^2} \\ &= \frac{16}{3\sigma\rho C_p}\frac{\sigma^*}{k^*}\frac{T_\infty^3\partial^2 T}{\partial y^2} \end{aligned}$$

Substitute into energy equation (3):

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\alpha + \frac{16}{3\sigma\rho C_p}\frac{\sigma^*}{k^*}T_\infty^3\right)\frac{\partial^2 T}{\partial y^2} + \frac{v}{C_p}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] + \frac{1}{\sigma\rho C_p}B_o^2u^2 \quad (4)$$

The boundary conditions may be expressed as:

$$\text{At } y = 0, T = T_w(x) = T_\infty + Ax^n.$$

$$\text{At } y \rightarrow \infty, u = U, T = T_\infty.$$

Using similarity transformation satisfying the continuity equation:

$$\eta = y\sqrt{\frac{U}{vx}}, \quad \psi = \sqrt{Uvx}f(\eta), \quad u = \frac{\partial\psi}{\partial y} = Uf' \quad \text{and} \quad v = -\frac{\partial\psi}{\partial x} = \frac{1}{2}\sqrt{\frac{Uv}{x}}(\eta f' - f)$$

With the use of usual chain and product rules of differentiation, eqn (2) and (3) reduce to non-linear ODEs:

$$\frac{5}{2}f'''(\eta) + d_1f''(\eta) + f(\eta)f''(\eta) - M(d_2f'(\eta) + \eta f'(\eta)f''(\eta) - f(\eta)f''(\eta)) = 0 \quad (5)$$

$$(1 + R_d)\theta''(\eta) = nP_r f'(\eta)\theta(\eta) - \frac{1}{2}P_r f(\eta)\theta'(\eta) - \frac{5}{4}P_r E_c (f''(\eta))^2 - \frac{1}{2\sigma^2}P_r E_c M(f'(\eta))^2 \quad (6)$$

Transformed BC are:

$$\text{at } \eta = 0 \quad \{f(0) = 0, f'(0) = 0, \theta(0) = 1\}$$

$$\text{at } \eta \rightarrow \infty \quad \{f'(\infty) = 1, \theta(\infty) = 0\} \quad (7)$$

for $E_c = \frac{U^2}{C_p(T_w - T_\infty)}$, $P_r = \frac{v}{\alpha}$, $d_1 = \frac{3}{2}\sqrt{\frac{v}{Ux}}$, $d_2 = \frac{2x}{U}$ and M is the magnetic parameter. Our target is to study how controlling factors E_c , P_r , n and M affect the rate of heat transfer.

SOLUTION METHODOLOGY

The PDEs are transformed into ODEs through the use of a similarity transformation and solved by the shooting method. Equations (5-6) are treated by using the bvp4c tool in Maple with BC specified in (7). Initially, the nonlinear eqns (5-7) were expressed as a system of linear equations:

$$f = y_1, f' = y_2, f'' = y_3, f''' = (yy)_3 \quad (8)$$

$$(yy)_3 = \frac{2M(d_2y_1 + \eta y_2 y_3 - y_1 y_3)}{5} \quad (9)$$

$$\text{and } \theta = y_4, \theta' = y_5, \theta'' = (yy)_4 \quad (10)$$

$$(yy)_4 = \frac{P_r}{1+R_d} \left(n y_2 y_4 - \frac{1}{2} y_1 y_5 - \frac{5}{4} E_c (y_3)^2 - \frac{1}{2\sigma^2} E_c M (y_2)^2 \right) \quad (11)$$

BC:

$$y_1(0) = 0, y_2(0) = 0, y_4(0) = 1, y_2(\infty) = 1 \text{ and } y_5(\infty) = 0.$$

DISCUSSION ON RESULTS

As shown in Fig. 1, the temperature profile decreases for $n > 0$, while keeping other parameters constant. As n increases, the fluid becomes thermally cooler at all locations, indicating reduced temperature distribution and a thinner thermal boundary layer.

In Fig.2, the temperature profile increases as the Eckert number increases for $n=1$. As the same happens in Fig.3, given $n=3$. An increase in Eckert number enhances viscous dissipation, converting more kinetic energy into heat, thereby increasing the fluid temperature and thickening the thermal boundary layer.

As shown in Fig.4, the temperature profile (for $n=1$) increases as the Prandtl number also increases. An increase in the Prandtl number reduces thermal diffusivity, causing heat to diffuse slowly and accumulate near the surface, thereby thinning the thermal boundary layer and increasing the temperature profile.

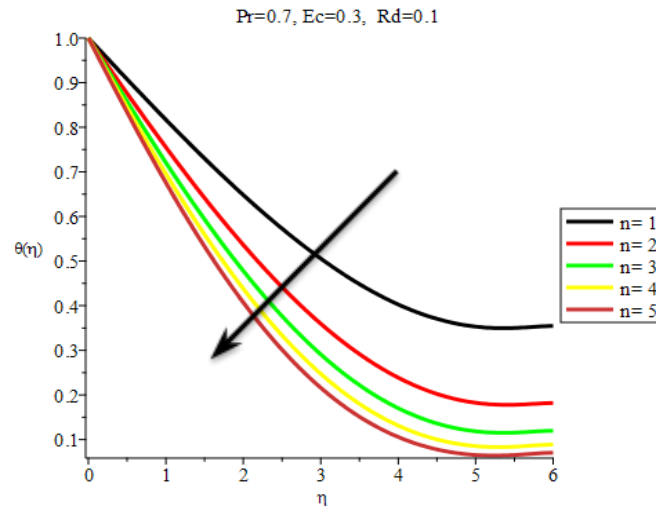


Fig.1. Impression of n on the temperature distribution $\theta(\eta)$.

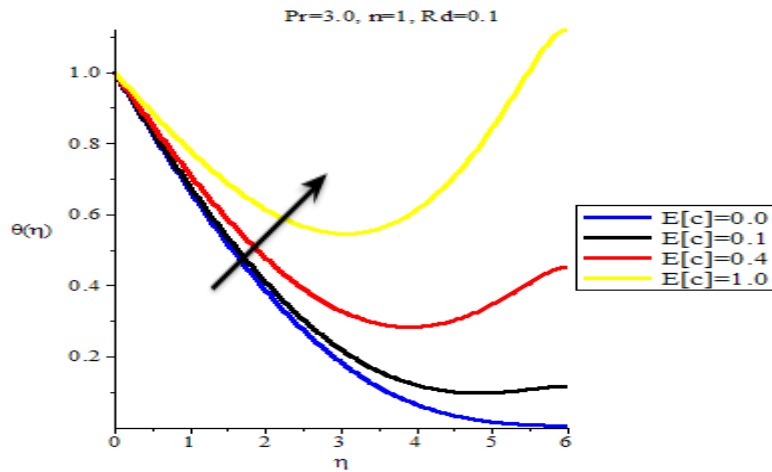


Fig.2. Impression of Ec on the temperature distribution $\theta(\eta)$.

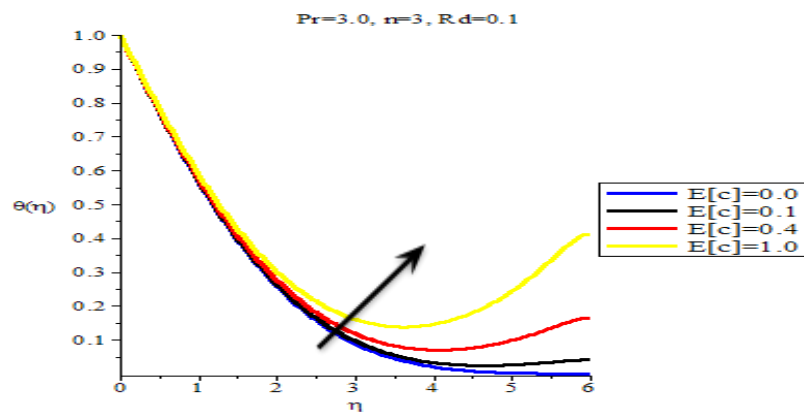


Fig.3. Impression of Ec on the temperature distribution $\theta(\eta)$.

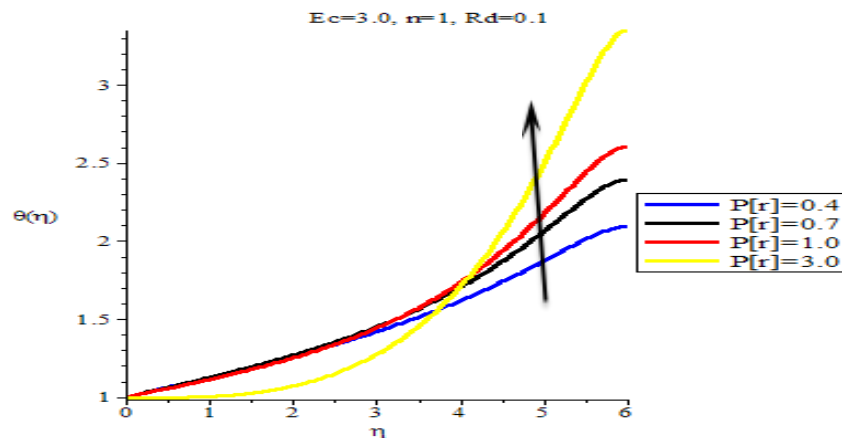


Fig.4. Impression of Pr on the temperature distribution $\theta(\eta)$.

In Fig. 5, there is an inverse relation between temperature profile and Prandtl number, taken $n=3$ and keeping Eckert number (Ec) and Radiation parameter Rd constant. For $n=3$, a decrease in the Prandtl number enhances thermal diffusivity, allowing heat to penetrate deeper into the fluid and resulting in an increase in the temperature profile throughout the boundary layer. However, there is a point shared irrespective of Pr .

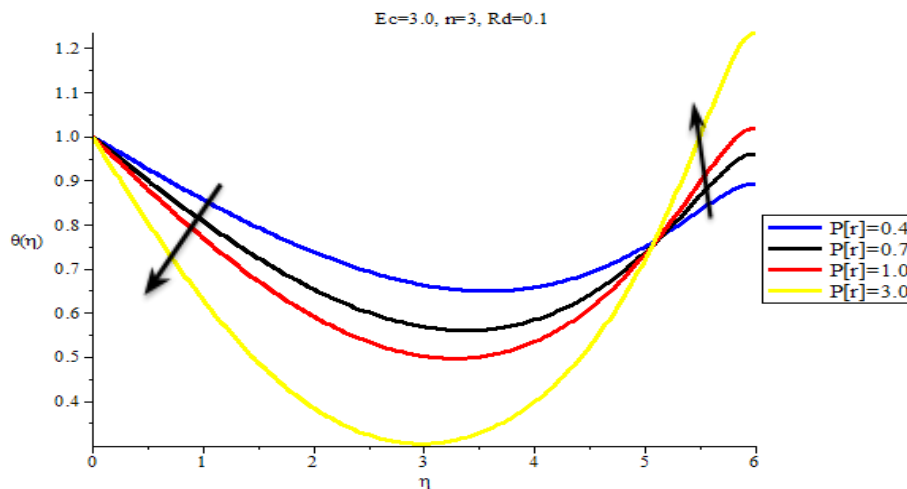


Fig.5. Impression of Pr on the temperature distribution $\theta(\eta)$.

Fig. 6 shows that the temperature profile and radiation term Rd are in inverse proportion as $n=1$. For $n=1$, a reduction in the radiation parameter weakens radiative heat loss, causing thermal energy to accumulate in the fluid and leading to an increase in the temperature profile across the boundary layer.

Taking $n=3$, the temperature profile and radiation term Rd are in direct relation as shown in Fig. 7. For $n=3$, an increase in the radiation parameter enhances radiative heat transfer, supplying additional thermal energy to the fluid and resulting in an increase in the temperature profile and thermal boundary-layer thickness. There exists a common point of temperature profile irrespective of Rd . But they are also inverse at some points.

The behaviour of the magnetic parameter M on the velocity profile $f(\eta)$, as in Fig.8, shows that the fluid velocity decreases as M increases. An increase in the magnetic parameter intensifies the Lorentz force, which acts as a resistive drag on the electrically conducting fluid, thereby suppressing the fluid motion and leading to a decrease in the velocity profile.

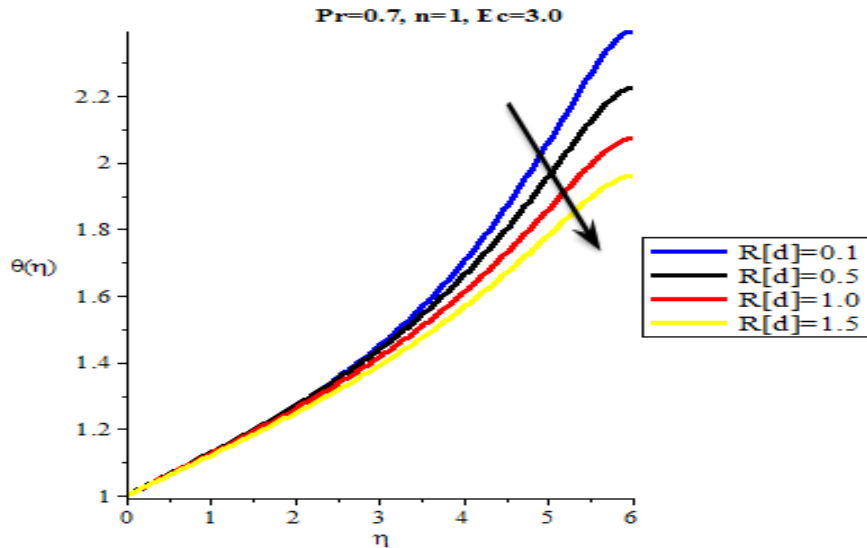


Fig.6. Impression of R_d on the temperature distribution $\theta(\eta)$.

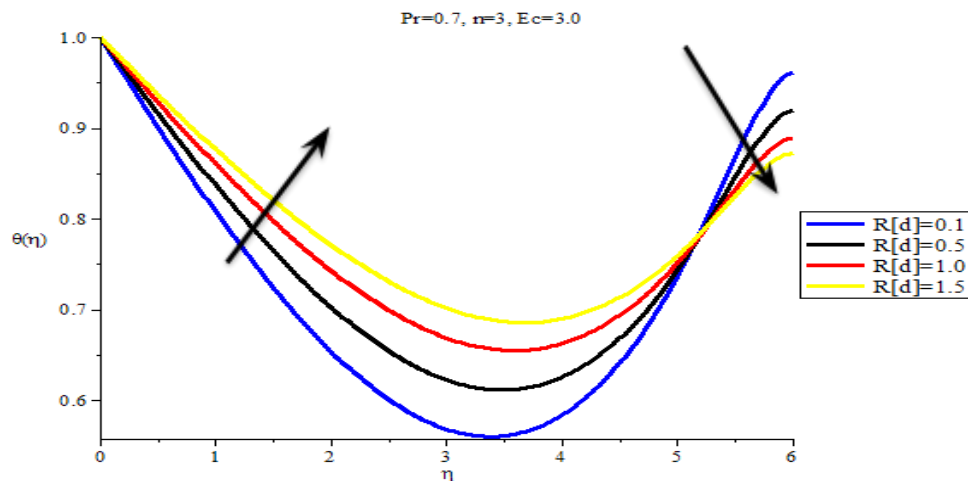


Fig.7. Impression of R_d on the temperature distribution $\theta(\eta)$.

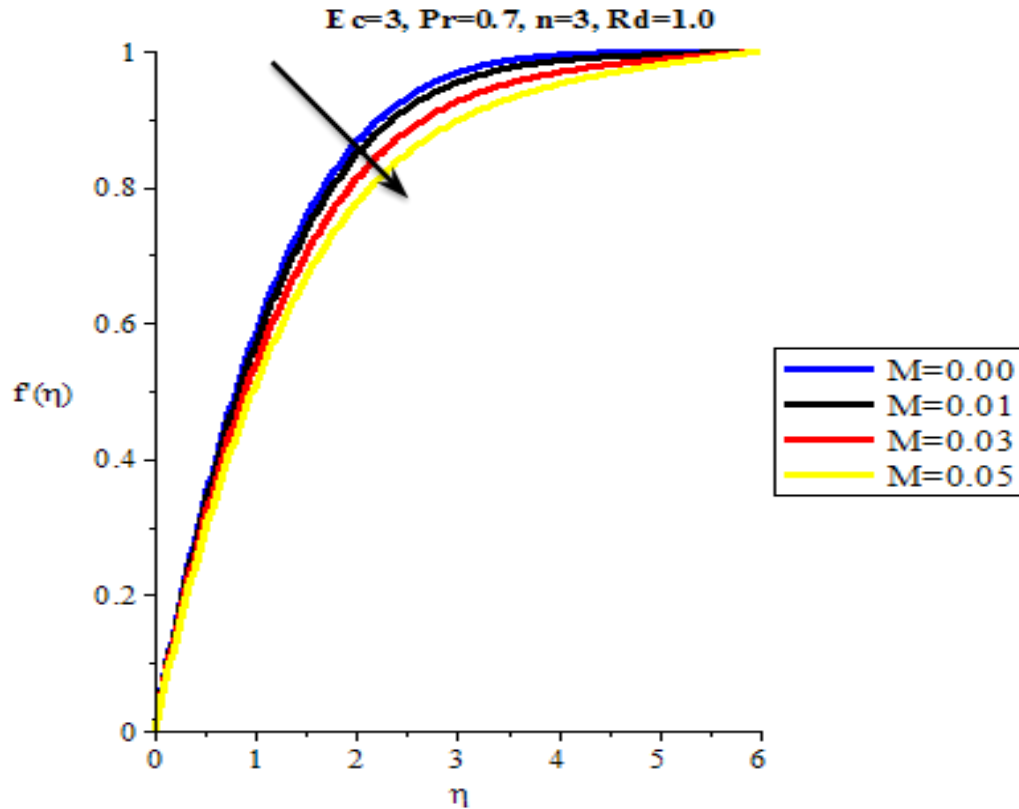


Fig.8. Impression of M on $f'(\eta)$ along a flat plate.

Numerical values for $\theta'(0)$ the wall are shown in Tables 1-3. In Table 1, $-\theta'(0)$ reduces as Ec increases for each $n = 1-4$. With Prandtl and Eckert numbers held constant, increasing n steepens the wall temperature gradient, indicating enhanced surface heat transfer and an increase in the local Nusselt number. With Prandtl number and n held constant, increasing the Eckert number enhances viscous dissipation, raising the fluid temperature near the wall, which reduces the wall temperature gradient and decreases the surface heat-transfer rate. This implies that the Nusselt number decreases.

Table 1: $Pr = 0.7$

n	$Ec = 0.1$	$Ec = 0.3$	$Ec = 0.5$	$Ec = 0.7$
1	0.05567691	0.05486335	0.05404979	0.05336220
2	0.05609889	0.05531085	0.05452281	0.05373477
3	0.05650924	0.05574564	0.05498204	0.05421844
4	0.05690857	0.05616837	0.05542818	0.05468799

From Table 2, Nu obviously increases when $n = 4$. With Eckert and Prandtl numbers held constant, increasing n steepens the wall temperature gradient, indicating enhanced heat

transfer and a higher local Nusselt number due to a thinner thermal boundary layer. With Eckert number and n constant, increasing the Prandtl number reduces thermal diffusivity, which slows heat transfer from the wall, resulting in a smaller wall temperature gradient and decreased surface heat-transfer rate.

Table 2: $Ec = 0.5$

n	$Pr = 0.7$	$Pr = 3$	$Pr = 5$
1	0.05404979	0.04978832	0.04680350
2	0.05452281	0.05225098	0.05128845
3	0.05498204	0.05442365	0.05495864
4	0.05542818	0.05635964	0.05803047

Table 3: $n = 3$

Ec	$Pr = 0.7$	$Pr = 3$	$Pr = 5$
0.1	0.05650924	0.05934844	0.06152813
0.3	0.05574564	0.05688605	0.05824339
0.5	0.05498204	0.05442365	0.05495864
0.7	0.05421844	0.05196126	0.05167390

In Table 3, Nu increases as the $-\theta'(0)$ number increases. With Prandtl number and n constant, increasing the Eckert number enhances viscous dissipation, raising the fluid temperature near the wall, which reduces the wall temperature gradient and decreases the surface heat-transfer rate.

With Eckert number and n held constant, increasing the Prandtl number reduces thermal diffusivity, causing heat to be confined near the wall, which steepens the wall temperature gradient and enhances surface heat transfer.

CONCLUSION

The influence of viscous dissipation, radiation and Joule heating on heat transfer over a flat plate is investigated by using the MHD boundary layer equations. The following has been observed:

i) The temperature profile for variable temperature increases as the Eckert number increases. It can be concluded that Viscous dissipation effects are strong. Mechanical energy is being converted into thermal energy, and the fluid temperature rises even without additional external heating. Temperature gradients near the wall may increase.

ii) for $n > 1$, as thermal radiation parameter (R_d) increases, there is an improvement in heat diffusivity, which has a positive effect on temperature distribution. This leads to an increase in the rate of energy transport within the fluid and raises the effective thermal diffusivity of the medium.

iii) as Ec increases, the rate of heat transfer at the wall ($-\theta'(0)$) reduces. For highly viscous flows, increasing the Eckert number can reduce cooling effectiveness, lower surface heat flux, and be critical in the thermal design of high-velocity systems.

Declaration of competing interest

No known competing interest.

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